BHARAT BHAWAN (P & D)

Mathematics

For Class 6

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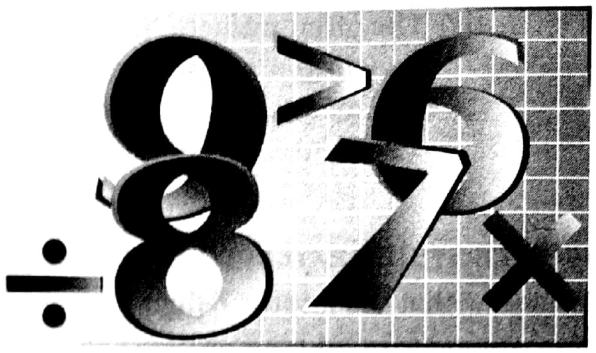
Contents

1 Number System



Numbers play an important role in mathematics. We have studied about counting numbers in primary classes. However, we shall review them here and extend our study to have the ideas of larger numbers.

We may express numbers in figures as well as in words.

NOTATION *Writing a number in figures is called notation.*

NUMERATION *Writing a number in words is called numeration.*

HINDU-ARABIC SYSTEM OF WRITING NUMBERS

In the Hindu-Arabic system, we use ten symbols, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, called ***digits*** or ***figures***, to represent any number.

A group of digits, denoting a number, is called a ***numeral.***

We use place-value system to represent a number.

For a given numeral, we start from the extreme right as: *Ones, Tens, Hundreds, Thousands, Ten thousands, Lakhs, Ten lakhs, etc.*

EXAMPLE 1. *Given below are some numbers arranged in a place-value table. Write each number in words and put it in the expanded form.*

PERIOD →	Lakhs		Thousands		Ones		
	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
(i)				9	6	3	8
(ii)			8	2	7	6	5
(iii)		3	5	6	8	0	4
(iv)	5	9	7	3	4	8	2

Solution (i) The given numeral is **'9638'**.
It is '*nine thousand six hundred thirty-eight*'.
In expanded form, we write it as:
$9638 = (9 \times 1000) + (6 \times 100) + (3 \times 10) + (8 \times 1)$.

(ii) The given numeral is **'82765'**.
It is '*eighty-two thousand seven hundred sixty-five*'.
In expanded form, we write it as:
$82765 = (8 \times 10000) + (2 \times 1000) + (7 \times 100) + (6 \times 10) + (5 \times 1)$.

(iii) The given numeral is **'356804'**.
It is '*three lakh fifty-six thousand eight hundred and four*'.
In expanded form, we write it as:
$356804 = (3 \times 100000) + (5 \times 10000) + (6 \times 1000) + (8 \times 100) + (0 \times 10) + (4 \times 1)$.

(iv) The given numeral is **'5973482'**.
It is *'fifty-nine lakh seventy-three thousand four hundred eighty-two'*.
In expanded form, we write it as:

$$5973482 = (5 \times 1000000) + (9 \times 100000) + (7 \times 10000) + (3 \times 1000) + (4 \times 100) + (8 \times 10) + (2 \times 1).$$

NUMBER SYSTEM EXTENDED FURTHER

We know that the largest 7-digit number is **9999999.**
It is *ninety-nine lakh ninety-nine thousand nine hundred ninety-nine.*
On adding 1 to it, we get

$(9999999 + 1) =$ **10000000,** called ***one crore.***

Thus, the smallest 8-digit number is one crore.
The largest 8-digit number is **99999999.**
It is *nine crore ninety-nine lakh ninety-nine thousand nine hundred ninety-nine.*
Thus, we extend our place-value chart to have

Ones, Tens, Hundreds, Thousands, Ten thousands, Lakhs, Ten lakhs, Crores and Ten crores, etc.

EXAMPLE 2. *Given below are the numerals*
(i) 75624908 *and (ii)* 853172069.
Put them in the place-value chart. Write them in words.
Put each one of them in the expanded form.

Solution The new place-value chart is given below.

PERIOD →	Crores		Lakhs		Thousands		Ones		
	Ten crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
(i)		7	5	6	2	4	9	0	8
(ii)	8	5	3	1	7	2	0	6	9

(i) The given numeral is **75624908.**
It is *seven crore fifty-six lakh twenty-four thousand nine hundred eight.*
Its expanded form is:

$$75624908 = (7 \times 10000000) + (5 \times 1000000) + (6 \times 100000) + (2 \times 10000) + (4 \times 1000) + (9 \times 100) + (0 \times 10) + (8 \times 1).$$

(ii) The given numeral is **853172069.**
It is *eighty-five crore thirty-one lakh seventy-two thousand sixty-nine.*
Its expanded form is:

$$853172069 = (8 \times 100000000) + (5 \times 10000000) + (3 \times 1000000) + (1 \times 100000) + (7 \times 10000) + (2 \times 1000) + (0 \times 100) + (6 \times 10) + (9 \times 1)$$

PERIODS *Crores, lakhs, thousands and hundreds are known as periods.*

SEPARATING PERIODS IN HINDU-ARABIC SYSTEM

The various periods are:

(Crores), (Lakhs), (Thousands), (Hundreds + Tens + Ones)

We insert comma after each period.

EXAMPLE 1. *Write 75624908, separating periods.*

Solution Separating periods, we have

Crores	Lakhs	Thousands	Ones		
			H	T	O
7	56	24	9	0	8

Using commas, we write it as **7,56,24,908.**

EXAMPLE 2. *Write 853172069, separating periods.*

Solution Separating periods, we have

Crores	Lakhs	Thousands	Ones		
			H	T	O
85	31	72	0	6	9

Using commas, we write it as **85,31,72,069.**

FACE VALUE OF A DIGIT IN A NUMERAL

The face value of a digit remains as it is, whatever place it may be occupying in the place-value chart.

Thus, the face value of 4 is always 4, wherever it may be.

PLACE VALUE OF A DIGIT IN A NUMERAL

The place value of a digit in a numeral depends upon the place it occupies in the place-value chart.

If 5 occurs at ones place, its place value = 5 ones = $(5 \times 1) = 5$.

If 5 occurs at tens place, its place value = 5 tens = $(5 \times 10) = 50$.

If 5 occurs at hundreds place, its place value = 5 hundreds = $(5 \times 100) = 500$, and so on.

Some more examples are given below.

EXAMPLE *Consider the numeral* **'72934806'.**

In this numeral, we have

place value of 6 = 6 ones = $(6 \times 1) = 6$;

place value of 0 = 0 tens = $(0 \times 10) = 0$;

place value of 8 = 8 hundreds = $(8 \times 100) = 800$;

place value of 4 = 4 thousands = $(4 \times 1000) = 4000$;

place value of 3 = 3 ten thousands = $(3 \times 10000) = 30000$;

place value of 9 = 9 lakhs = $(9 \times 100000) = 900000$;

place value of 2 = 2 ten lakhs = $(2 \times 1000000) = 2000000$;

place value of 7 = 7 crores = $(7 \times 10000000) = 70000000$.

SOLVED EXAMPLES

EXAMPLE 1. *Separate the periods of the numeral '93574862' by commas and write it in words.*

Solution Separating periods, we have

Crores	Lakhs	Thousands	Ones		
			H	T	O
9	35	74	8	6	2

Using commas, we write it as **9,35,74,862.**
In words, we write it as:
'*nine crore thirty-five lakh seventy-four thousand eight hundred sixty-two*'.

EXAMPLE 2. *Find the difference of the place values of the two 7s in 75810764.*

Solution The place value of 7 at crores place = 7 crores = $(7 \times 10000000) = 70000000$.
The place value of 7 at hundreds place = 7 hundreds = $(7 \times 100) = 700$.
Required difference = $(70000000 - 700) = 69999300$.

EXAMPLE 3. *How many 5-digit numbers are there in all?*

Solution The largest 5-digit number = 99999.
The smallest 5-digit number = 10000.
Number of all 5-digit numbers = $(99999 - 10000) + 1$
$= (89999 + 1) = 90000$.
Hence, the number of all 5-digit numbers is ninety thousand.

EXAMPLE 4. *How many 8-digit numbers are there in all?*

Solution The largest 8-digit number = 99999999.
The smallest 8-digit number = 10000000.
Number of all 8-digit numbers = $(99999999 - 10000000) + 1$
$= (89999999 + 1) = 90000000$
= nine crores.
Hence, there are in all nine crores of 8-digit numbers.

EXAMPLE 5. *Write the smallest 8-digit number having four different digits.*

Solution Four smallest digits are 0, 1, 2, 3.
Hence, the required number is 10000023.

EXAMPLE 6. *Write all 3-digit numbers using the digits 1, 3, 5, taking each digit only once.*

Solution Keeping 1 at the ones place, the numbers formed are 351 and 531.
Keeping 3 at the ones place, the numbers formed are 153 and 513.
Keeping 5 at the ones place, the numbers formed are 135 and 315.
So, the required numbers are
351, 531, 153, 513, 135 and 315.

INTERNATIONAL SYSTEM OF NUMERATION

In the international system of numeration adopted by all the countries throughout the world, the place-value chart is as follows.

PERIOD →	Billions		Millions			Thousands			Ones		
	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
				3	4	1	2	5	6	8	0
			TC	C	TL	L	T Th	Th	H	T	O

The number '34125680' in the international system will be read as

'thirty-four million one hundred and twenty-five thousand six hundred eighty'.

It is clear from the table that the above number in the Hindu-Arabic system is given by

'three crore forty-one lakh twenty-five thousand six hundred eighty'.

EXAMPLE 7. *Rewrite each of the following numbers with proper commas, using international place-value chart:*

(i) 53684 *(ii)* 1286475 *(iii)* 49637582

Also, write the number name of each in the international system.

Solution Let us arrange the given numerals in an international place-value chart. Then separating the periods, we write them as shown below.

	Millions			Thousands			Ones			
	HM	TM	M	H Th	T Th	Th	H	T	O	
(i)					5	3	6	8	4	53,684
(ii)			1	2	8	6	4	7	5	1,286,475
(iii)		4	9	6	3	7	5	8	2	49,637,582
		C	**TL**	**L**	**T Th**	**Th**	**H**	**T**	**O**	

In the international system, we write them in words as:

(i) Fifty-three thousand six hundred eighty-four

(ii) One million two hundred eighty-six thousand four hundred seventy-five

(iii) Forty-nine million six hundred thirty-seven thousand five hundred eighty-two

In the Hindu-Arabic system, these numbers are:

(i) Fifty-three thousand six hundred eighty-four

(ii) Twelve lakh eighty-six thousand four hundred seventy-five

(iii) Four crore ninety-six lakh thirty-seven thousand five hundred eighty-two

EXERCISE 1A

1. Write the numeral for each of the following numbers:
   (i) Nine thousand eighteen
   (ii) Fifty-four thousand seventy-three
   (iii) Three lakh two thousand five hundred six
   (iv) Twenty lakh ten thousand eight
   (v) Six crore five lakh fifty-seven
   (vi) Two crore two lakh two thousand two hundred two
   (vii) Twelve crore twelve lakh twelve thousand twelve
   (viii) Fifteen crore fifty lakh twenty thousand sixty-eight
2. Write each of the following numbers in words:
   (i) 63,005 (ii) 7,07,075 (iii) 34,20,019
   (iv) 3,05,09,012 (v) 5,10,03,604 (vi) 6,18,05,008
   (vii) 19,09,09,900 (viii) 6,15,30,807 (ix) 6,60,60,060
3. Write each of the following numbers in expanded form:
   (i) 15,768 (ii) 3,08,927 (iii) 24,05,609
   (iv) 5,36,18,493 (v) 6,06,06,006 (vi) 9,10,10,510

4. Write the corresponding numeral for each of the following:
   (i) $6 \times 10000 + 2 \times 1000 + 5 \times 100 + 8 \times 10 + 4 \times 1$
   (ii) $5 \times 100000 + 8 \times 10000 + 1 \times 1000 + 6 \times 100 + 2 \times 10 + 3 \times 1$
   (iii) $2 \times 10000000 + 5 \times 100000 + 7 \times 1000 + 9 \times 100 + 5 \times 1$
   (iv) $3 \times 1000000 + 4 \times 100000 + 6 \times 1000 + 5 \times 100 + 7 \times 1$
5. Find the difference between the place values of the two nines in 79520986.
6. Find the difference between the place value and the face value of 7 in 27650934.
7. How many 6-digit numbers are there in all?
8. How many 7-digit numbers are there in all?
9. How many thousands make a lakh?
10. How many thousands make a crore?
11. Find the difference between the number 738 and that obtained on reversing its digits.
12. What comes just after 9547999?
13. What comes just before 9900000?
14. What comes just before 10000000?
15. Write all 3-digit numbers using 2, 3, 4, taking each digit only once.
16. Write the smallest number of different digits formed by using the digits 3, 1, 0, 5 and 7.
17. Write the largest number of different digits formed by using the digits 2, 4, 0, 3, 6 and 9.
18. Rewrite each of the following numerals with proper commas, using the international place-value chart. Also, write the number name of each in the international system.
   (i) 735821 (ii) 6057894 (iii) 56943821
   (iv) 37502093 (v) 89350064 (vi) 90703006
19. Write each of the following in figures in the international place-value chart:
   (i) Thirty million one hundred five thousand sixty-three
   (ii) Fifty-two million two hundred five thousand six
   (iii) Five million five thousand five

COMPARISON OF NUMBERS

In order to compare two numbers, we adopt the following rules.

Rule 1. *The number with less digits is less than the number with more digits.*

Rule 2. *Suppose we have to compare two numbers having the same number of digits, then we proceed as under.*

Step 1. *First compare the digits at the leftmost place in both the numbers.*

Step 2. *If they are equal in value then compare the second digits from the left.*

Step 3. *If the second digits from the left are equal then compare the third digits from the left.*

Step 4. *Continue until you come across unequal digits at the corresponding places. Clearly, the number with greater such digit is the greater of the two.*

The following examples will make the ideas more clear.

SOLVED EXAMPLES

EXAMPLE 1. *Which is greater:* 24576813 *or* 9897686?

Solution Here, we have to compare 24576813 and 9897686.
Clearly, 24576813 consists of 8 digits while 9897686 contains 7 digits.
We know that a number with more digits is greater.
∴ 24576813 > 9897686.

EXAMPLE 2. *Which is greater:* 96850374 *or* 96721895?

Solution Let us arrange the given numbers in a place-value chart, as shown below.

Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
9	6	8	5	0	3	7	4
9	6	7	2	1	8	9	5

Clearly, both the numbers have 8 digits.
At the crores place both have the same digit, namely, 9.
At the ten lakhs place both have the same digit, namely, 6.
But, at the lakhs place, the first number has 8 while the second has 7.
Clearly, 8 > 7.
∴ 96850374 > 96721895.

EXAMPLE 3. *Arrange the following numbers in ascending order:*
3763214, 18340217, 984671, 3790423, 18431056

Solution Let us arrange the given numbers in a place-value chart, as shown below.

Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
	3	7	6	3	2	1	4
1	8	3	4	0	2	1	7
		9	8	4	6	7	1
	3	7	9	0	4	2	3
1	8	4	3	1	0	5	6

Out of the given numbers one is a 6-digit number, two are 7-digit numbers and two are 8-digit numbers.
6-digit number is 984671.
In 7-digit numbers we have 3763214 < 3790423.
In 8-digit numbers we have 18340217 < 18431056.
Hence, the given numbers in ascending order are:
984671 < 3763214 < 3790423 < 18340217 < 18431056.

EXAMPLE 4. *Arrange the following numbers in descending order:*
63872604, 4965328, 63890503, 5023145, 576943

Solution Let us arrange the given numbers in a place-value chart.

Ten crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
	6	3	8	7	2	6	0	4
		4	9	6	5	3	2	8
	6	3	8	9	0	5	0	3
		5	0	2	3	1	4	5
			5	7	6	9	4	3

In 8-digit numbers, clearly 63890503 > 63872604 (9 T Th > 7 T Th).
In 7-digit numbers, clearly 5023145 > 4965328 (5 TL > 4 TL).
The 6-digit number is 576943, which is clearly the smallest.
∴ 63890503 > 63872604 > 5023145 > 4965328 > 576943.
Hence, the given numbers in descending order are:
63890503, 63872604, 5023145, 4965328, 576943.

EXERCISE 1B

Fill in each of the following boxes with the correct symbol > or <:

1. 1003467 ☐ 987965
2. 3572014 ☐ 10235401
3. 3254790 ☐ 3260152
4. 10357690 ☐ 11243567
5. 27596381 ☐ 7965412
6. 47893501 ☐ 47894021

Arrange the following numbers in descending order:

7. 63521047, 7354206, 63514759, 7355014, 102345680
8. 5032786, 23794206, 5032790, 23756819, 987876
9. 190909, 1808088, 16060666, 16007777, 181888, 1808090
10. 199988, 1704382, 200175, 1702497, 201200, 1712040

Arrange the following numbers in ascending order:

11. 9873426, 24615019, 990357, 9874012, 24620010
12. 56943201, 5694437, 56944000, 5695440, 56943300
13. 700087, 8014257, 8015032, 10012458, 8014306
14. 1020304, 893245, 980134, 1021403, 893425, 1020216

WORD PROBLEMS ON NUMBER OPERATIONS

ILLUSTRATIVE EXAMPLES

EXAMPLE 1. *A businessman earned ₹ 3648970 in the year 2014. Next year, his earning was increased by ₹ 956880. What was his earning in the year 2015?*

Solution Earning of the man in the year 2014 = ₹ 3648970
Increase in earning during next year = ₹ 956880
Man's earning in the year 2015 = ₹ (3648970 + 956880) = ₹ 4605850.

	1 TL	1 L	1 T Th	1 Th	1 H	T	O
	3	6	4	8	9	7	0
+		9	5	6	8	8	0
	4	6	0	5	8	5	0

Hence, the man's earning in the year 2015 is ₹ 4605850.

EXAMPLE 2. *A survey shows that the population of Andhra Pradesh is 98306965, that of Karnataka is 89627598 and that of Kerala is 46308927. What is the total population of these three states?*

Solution We have:
Population of Andhra Pradesh = 98306965.
Population of Karnataka = 89627598.
Population of Kerala = 46308927.
Total population of the three states = (98306965 + 89627598 + 46308927)

	2	2	1		2	2	1	2	
	TC	C	TL	L	T Th	Th	H	T	O
		9	8	3	0	6	9	6	5
+		8	9	6	2	7	5	9	8
+		4	6	3	0	8	9	2	7
	2	3	4	2	4	3	4	9	0

The total population of the three states is 234243490.

EXAMPLE 3. *The difference between two numbers is* 9476583. *If the smaller number is* 6873547, *find the greater number.*

Solution We have:
Difference between the two numbers = 9476583.
Smaller number = 6873547.
Hence, greater number = (9476583 + 6873547)

	1	1	1	1	1	1	1	
	C	TL	L	T Th	Th	H	T	O
		9	4	7	6	5	8	3
+		6	8	7	3	5	4	7
	1	6	3	5	0	1	3	0

Hence, the greater number is 16350130.

EXAMPLE 4. *The population of a city in the year* 2014 *was* 14693675. *In the following year, the population became* 18002403. *Find the increase in the population.*

Solution We have:
Population in the year 2015 = 18002403.
Population in the year 2014 = 14693675.
Increase in population = (18002403 – 14693675)

	C	TL	L	T Th	Th	H	T	O
	1	8	0	0	2	4	0	3
–	1	4	6	9	3	6	7	5
		3	3	0	8	7	2	8

Hence, the increase in population is 3308728.

EXAMPLE 5. *There was a stock of* 17380245 *quintals of wheat in a godown of the Food Corporation of India. Out of this stock,* 2756744 *quintals of wheat was sent to Haryana and* 4863108 *quintals to Punjab. How much is the balance stock now?*

Solution We have:
Total stock of wheat = 17380245 quintals.
Quantity of wheat sent to Haryana
= 2756744 quintals.
Quantity of wheat sent to Punjab
= 4863108 quintals.
Total quantity of wheat taken out of the godown
= (2756744 + 4863108) quintals
= 7619852 quintals.

		1	1				1	
		2	7	5	6	7	4	4
+	4	8	6	3	1	0	8	
	7	6	1	9	8	5	2	

	1	7	3	8	0	2	4	5
–	7	6	1	9	8	5	2	
	9	7	6	0	3	9	3	

Balance stock of wheat in the godown = (17380245 – 7619852) quintals
= 9760393 quintals.

EXAMPLE 6. *The cost of a steel almirah is ₹ 22875. What is the cost of 465 such almirahs?*

Solution Cost of 1 almirah = ₹ 22875.
Cost of 465 almirahs = ₹ (22875 × 465)
= ₹ 10636875.
Hence, the cost of 465 almirahs is ₹ 10636875.

```
    2 2 8 7 5
      × 4 6 5
  1 1 4 3 7 5
1 3 7 2 5 0 x
9 1 5 0 0 x x
1 0 6 3 6 8 7 5
```

EXAMPLE 7. *6895 screws can be packed in one carton. How many screws can be packed in 1685 such cartons?*

Solution We have:
Number of screws in 1 carton
= 6895.
Number of screws in 1685 cartons
= (6895 × 1685)
= 11618075.
Hence, the number of screws in 1685 cartons is 11618075.

```
      6 8 9 5
    × 1 6 8 5
    3 4 4 7 5
  5 5 1 6 0 x
4 1 3 7 0 x x
6 8 9 5 x x x
1 1 6 1 8 0 7 5
```

EXAMPLE 8. *The mass of each gas cylinder is 16 kg 250 g. What is the total mass of 18 such cylinders?*

Solution Mass of 1 cylinder = 16 kg 250 g.
Mass of 18 cylinders = (16 kg 250 g) × 18
= 292 kg 500 g.
Hence, the total mass of 18 cylinders is 292 kg 500 g.

```
(+4)
kg    g
16    250
      × 18
292   500
```

EXAMPLE 9. *The piece of cloth required for a shirt is 2 m 85 cm. How much cloth will be required for 16 such shirts?*

Solution We have:
Cloth required for 1 shirt = 2 m 85 cm.
Cloth required for 16 shirts = (2 m 85 cm) × 16
= 45 m 60 cm.
Hence, the cloth required for 16 shirts = 45 m 60 cm.

```
(+13)
m     cm
2     85
      × 16
45    60
```

EXAMPLE 10. *The cost of 16 flats constructed by UP Awas and Vikas Parishad is ₹ 24809520. What is the cost of each flat?*

Solution We have:
Total cost of 16 flats = ₹ 24809520.
Cost of each flat = (₹ 24809520) ÷ 16
= ₹ 1550595.
Hence, the cost of each flat is ₹ 1550595.

```
16) 24809520 (1550595
 - 16
   88
 - 80
    80
 - 80
     95
   - 80
     152
   - 144
      80
    - 80
    ×
```

EXAMPLE 11. *For making 16 shirts, 44 metres of cloth is needed. How much cloth is required for each shirt?*

Solution We have:

Cloth required for 16 shirts = 44 m.

Cloth required for each shirt = (44 m) ÷ 16

= 2 m 75 cm.

Hence, the cloth required for each shirt

= 2 m 75 cm.

```
        2 m
16 ) 44 m
     - 32
      12 m
     × 100
16 ) 1200 cm ( 75 cm
   - 112
      80
    - 80
      ×
```

EXAMPLE 12. *A car covers 1002 km in 16 hours. At what speed per hour does the car move?*

Solution Distance covered in 16 hours = 1002 km.

Distance covered in 1 hour = (1002 km) ÷ 16

= 62 km 625 m.

Hence, the speed of the car is 62 km 625 m per hour.

```
16 ) 1002 km ( 62 km
   - 96
      42
    - 32
      10 km
    × 1000
16 ) 10000 m ( 625 m
   - 96
      40
    - 32
      80
    - 80
      ×
```

EXERCISE 1C

1. The number of persons who visited the holy shrine of Mata Vaishno Devi during last two consecutive years was 13789509 and 12976498 respectively. How many persons visited the shrine during these two years?
2. Last year, three sugar factories in a town produced 24809565 bags, 18738576 bags and 9564568 bags of sugar respectively. How many bags were produced by all the three factories during last year?
3. A number exceeds 37684955 by 3615045. What is that number?
4. There were three candidates in an election. They received 687905 votes, 495086 votes and 93756 votes respectively. The number of invalid votes was 13849. If 25467 persons did not vote, find how many votes were registered.
5. A survey conducted on an Indian state shows that 1623546 people have only primary education; 9768678 people have secondary education; 6837954 people have higher education and 2684536 people are illiterate. If the number of children below the age of school admission is 698781, find the total population of the state.
6. In a particular year a company produced 8765435 bicycles. Next year, the number of bicycles produced was 1378689 more than those produced in the preceding year.
   How many bicycles were produced during the second year?
   How many bicycles were produced during these two years?
7. The sale receipt of a company during a year was ₹ 20956480. Next year, it increased by ₹ 6709570. What was the total sale receipt of the company during these two years?
8. The total population of a city is 28756304. If the number of males is 16987059, find the number of females in the city.

9. By how much is 13246510 larger than 4658642?
10. By how much is 5643879 smaller than one crore?
11. What number must be subtracted from 11010101 to get 2635967?
12. The sum of two numbers is 10750308. If one of them is 8967519, what is the other number?
13. A man had ₹ 20000000 with him. He spent ₹ 13607085 on buying a school building. How much money is left with him?
14. A society needed ₹ 18536000 to buy a property. It collected ₹ 7253840 as membership fee, took a loan of ₹ 5675450 from a bank and collected ₹ 2937680 as donation. How much is the society still short of?
15. A man had ₹ 10672540 with him. He gave ₹ 4836980 to his wife, ₹ 3964790 to his son and the rest to his daughter. How much money was received by the daughter?
16. The cost of a chair is ₹ 1485. How much will 469 such chairs cost?
17. How much money was collected from 1786 students of a school for a charity show if each student contributed ₹ 625?
18. A factory produces 6985 screws per day. How many screws will it produce in 358 days?
19. Mr Bhaskar saves ₹ 8756 every month. How much money will he save in 13 years?
20. A scooter costs ₹ 36725. How much will 487 such scooters cost?
21. An aeroplane covers 1485 km in 1 hour. How much distance will it cover in 72 hours?
22. The product of two numbers is 13421408. If one of the numbers is 364, find the other.
23. If 36 flats cost ₹ 68251500, what is the cost of each such flat?
24. The mass of a cylinder filled with gas is 30 kg 250 g and the mass of the empty cylinder is 14 kg 480 g. How much is the mass of the gas contained in it?
25. From a cloth 5 m long, a piece of length 2 m 85 cm is cut off. What is the length of the remaining piece?
26. In order to make a shirt, a length of 2 m 75 cm of cloth is needed. How much length of the cloth will be required for 16 such shirts?
27. For making 8 trousers of the same size, 14 m 80 cm of cloth is needed. How much cloth will be required for each such trouser?
28. The mass of a brick is 2 kg 750 g. What is the total mass of 14 such bricks?
29. The total mass of 8 packets, each of the same size, is 10 kg 600 g. What is the mass of each such packet?
30. A rope of length 10 m has been divided into 8 pieces of the same length. What is the length of each piece?

ESTIMATION

Before estimation, we must know how to round off a number to the nearest ten, nearest hundred, nearest thousand, etc.

For the same, we need the rules given below:

Rounding a Number to the Nearest Ten

Step 1: See the *ones digit* of the given number.

Step 2: If ones digit is less than 5, *replace ones digit by* 0, *and keep the other digits as they are.*

Step 3: If ones digit is 5 or more, *increase tens digit by* 1, *and replace ones digit by* 0.

Rounding a Number to the Nearest Hundred

Step 1: See the *tens digit* of the given number.

Step 2: If tens digit is less than 5, *replace each one of tens and ones digits by 0, and keep the other digits as they are.*

Step 3: If this digit is 5 or more, *increase hundreds digit by 1 and replace each digit on its right by 0.*

Rounding a Number to the Nearest Thousand

Step 1: See the *hundreds digit* of the given number.

Step 2: If hundreds digit is less than 5, *replace each one of hundreds, tens and ones digits by 0, and keep the other digits as they are.*

Step 3: If hundreds digit is 5 or more, *increase thousands digit by 1 and replace each digit on its right by 0.*

We may extend the ideas for larger numbers.

SOLVED EXAMPLES

EXAMPLE 1. *Round each of the following numbers to the nearest ten:*
(a) 53 (b) 287 (c) 8364 (d) 2045

Solution
(a) In 53, the ones digit is $3 < 5$.
$\therefore$ the required rounded number = 50.
(b) In 287, the ones digit is $7 > 5$.
$\therefore$ the required rounded number = 290.
(c) In 8364, the ones digit is $4 < 5$.
$\therefore$ the required rounded number = 8360.
(d) In 2045, the ones digit is $5 = 5$.
$\therefore$ the required rounded number = 2050.

EXAMPLE 2. *Round each of the following numbers to the nearest hundred:*
(a) 648 (b) 2356 (c) 13768 (d) 1249

Solution
(a) In 648, the tens digit is $4 < 5$.
$\therefore$ the required rounded number = 600.
(b) In 2356, the tens digit is $5 = 5$.
$\therefore$ the required rounded number = 2400.
(c) In 13768, the tens digit is $6 > 5$.
$\therefore$ the required rounded number = 13800.
(d) In 1249, the tens digit is $4 < 5$.
$\therefore$ the required rounded number = 1200.

EXAMPLE 3. *Round each of the following numbers to the nearest thousand:*
(a) 5486 (b) 6823 (c) 14380 (d) 23659

Solution
(a) In 5486, the hundreds digit is $4 < 5$.
$\therefore$ the required rounded number = 5000.
(b) In 6823, the hundreds digit is $8 > 5$.
$\therefore$ the required rounded number = 7000.
(c) In 14380, the hundreds digit is $3 < 5$.
$\therefore$ the required rounded number = 14000.
(d) In 23659, the hundreds digit is $6 > 5$.
$\therefore$ the required rounded number = 24000.

Estimation

To estimate means to make a guess. Estimation, thus, gives us a rough idea of the answer to a question involving operations on numbers.

ESTIMATING THE SUMS

EXAMPLE 1. *Estimate the sum* (64 + 79) *to the nearest ten.*

Solution 64 estimated to the nearest ten = 60.
79 estimated to the nearest ten = 80.
Hence, the required estimation = (60 + 80) = 140.

EXAMPLE 2. *Estimate the sum* (267 + 132) *to the nearest ten.*

Solution 267 estimated to the nearest ten = 270.
132 estimated to the nearest ten = 130.
Hence, the required estimation = (270 + 130) = 400.

EXAMPLE 3. *Estimate the sum* (274 + 143) *to the nearest hundred.*

Solution 274 estimated to the nearest hundred = 300.
143 estimated to the nearest hundred = 100.
Hence, the required estimation = (300 + 100) = 400.

EXAMPLE 4. *Estimate the sum* (21397 + 27807 + 42505) *to the nearest thousand.*

Solution 21397 estimated to the nearest thousand = 21000.
27807 estimated to the nearest thousand = 28000.
42505 estimated to the nearest thousand = 43000.
Hence, the required estimation = (21000 + 28000 + 43000) = 92000.

EXAMPLE 5. *Estimate the difference* (673 – 258) *to the nearest hundred.*

Solution 673 estimated to the nearest hundred = 700.
258 estimated to the nearest hundred = 300.
Hence, the required estimation = (700 – 300) = 400.

EXERCISE 1D

1. Round each of the following numbers to the nearest ten:
(a) 36 (b) 173 (c) 3869 (d) 16378

2. Round each of the following numbers to the nearest hundred:
(a) 814 (b) 1254 (c) 43126 (d) 98165

3. Round each of the following numbers to the nearest thousand:
(a) 793 (b) 4826 (c) 16719 (d) 28394

4. Round each of the following numbers to the nearest ten thousand:
(a) 17514 (b) 26340 (c) 34890 (d) 272685

Estimate each sum to the nearest ten:

5. (57 + 34) **6.** (43 + 78) **7.** (14 + 69)
8. (86 + 19) **9.** (95 + 58) **10.** (77 + 63)
11. (356 + 275) **12.** (463 + 182) **13.** (538 + 276)

Estimate each sum to the nearest hundred:

14. (236 + 689) **15.** (458 + 324) **16.** (170 + 395)

17. (3280 + 4395) 18. (5130 + 1410) 19. (10083 + 29380)

Estimate each sum to the nearest thousand:

20. (32836 + 16466) 21. (46703 + 11375)

22. There are 54 balls in box A and 79 balls in box B. Estimate the total number of balls in both the boxes taken together.

Estimate each difference to the nearest ten:

23. (53 – 18) 24. (97 – 38) 25. (409 – 148)

Estimate each difference to the nearest hundred:

26. (678 – 215) 27. (957 – 578) 28. (7258 – 2429) 29. (5612 – 3095)

Estimate each difference to the nearest thousand:

30. (35863 – 27677) 31. (47005 – 39488)

ESTIMATING THE PRODUCTS

ILLUSTRATIVE EXAMPLES

EXAMPLE 1. *Estimate the product of 42 and 58.*

Solution 42 estimated to the nearest ten = 40.
58 estimated to the nearest ten = 60.
Hence, the required estimation = (40 × 60) = 2400.

EXAMPLE 2. *Estimate the product of 34 and 75.*

Solution 34 estimated to the nearest ten = 30.
75 estimated to the nearest ten = 80.
Hence, the required estimation = (30 × 80) = 2400.

EXAMPLE 3. *Estimate the product of 367 × 231 by rounding off each number to the nearest hundred.*

Solution 367 estimated to the nearest hundred = 400.
231 estimated to the nearest hundred = 200.
Hence, the estimated product = 400 × 200 = 80000.

EXAMPLE 4. *Estimate the product of 183 × 153 by rounding off the first number upwards and the second number downwards.*

Solution 183 estimated upwards = 200.
153 estimated downwards = 100.
Hence, the estimated product = 200 × 100 = 20000.

EXERCISE 1E

Estimate each of the following products by rounding off each number to the nearest ten:

1. 38 × 63 2. 54 × 47 3. 28 × 63
4. 42 × 75 5. 64 × 58 6. 15 × 34

Estimate each of the following products by rounding off each number to the nearest hundred:

7. 376×123 **8.** 264×147 **9.** 423×158
10. 509×179 **11.** 392×138 **12.** 271×339

Estimate each of the following products by rounding off the first number upwards and the second number downwards:

13. 183×154 **14.** 267×146 **15.** 359×76
16. 472×158 **17.** 680×164 **18.** 255×350

Estimate each of the following products by rounding off the first number downwards and the second number upwards:

19. 356×278 **20.** 472×76 **21.** 578×369

ESTIMATING THE QUOTIENTS

ILLUSTRATIVE EXAMPLE

EXAMPLE *Find the estimated quotient for each of the following:*
(i) $627 \div 23$ *(ii)* $985 \div 48$ *(iii)* $74 \div 34$ *(iv)* $694 \div 58$

Solution
(i) $627 \div 23$ is approximately equal to $600 \div 20 = 30$.
(ii) $985 \div 48$ is approximately equal to $1000 \div 50 = 20$.
(iii) $74 \div 34$ is approximately equal to $70 \div 30 = 7 \div 3$, which is approximately equal to 2.
(iv) $694 \div 58$ is approximately equal to $700 \div 60$,
which is approximately equal to $70 \div 6$,
which is approximately equal to 12.

EXERCISE 1F

Find the estimated quotient for each of the following:

1. $87 \div 28$ **2.** $83 \div 17$ **3.** $75 \div 23$
4. $193 \div 24$ **5.** $725 \div 23$ **6.** $275 \div 25$
7. $633 \div 33$ **8.** $729 \div 29$ **9.** $858 \div 39$
10. $868 \div 38$

ROMAN NUMERALS

ROMAN NUMERALS One of the early systems of writing numerals is the system of Roman numerals.

There are seven basic symbols to write any numeral.
These symbols are given below.

Roman numeral	I	V	X	L	C	D	M
Hindu-Arabic numeral	1	5	10	50	100	500	1000

If a bar is placed over a numeral, it is multiplied by 1000.

Thus, $\overline{V} = 5000$ and $\overline{X} = 10000$, etc.

Using these symbols, we may form all Roman numerals by adopting the rules given below.

RULE 1 *Repetition of a symbol in a Roman numeral means addition.*

CAUTIONS (i) Only I, X, C, M can be repeated.

(ii) V, L and D are never repeated.

(iii) No symbol in a Roman numeral can be repeated more than 3 times.

EXAMPLES (i) II = (1 + 1) = 2 (ii) XX = (10 + 10) = 20

(iii) XXX = (10 + 10 + 10) = 30 (iv) CC = (100 + 100) = 200

RULE 2 *A smaller numeral written to the right of a larger numeral is always added to the larger numeral.*

EXAMPLES (i) VI = (5 + 1) = 6 (ii) VIII = (5 + 1 + 1 + 1) = 8

(iii) XV = (10 + 5) = 15 (iv) LX = (50 + 10) = 60

RULE 3 *A smaller numeral written to the left of a larger numeral is always subtracted from the larger numeral.*

CAUTIONS (i) V, L and D are never subtracted.

(ii) I can be subtracted from V and X only.

(iii) X can be subtracted from L and C only.

(iv) C can be subtracted from D and M only.

EXAMPLES (i) IV = (5 – 1) = 4 (ii) IX = (10 – 1) = 9

(iii) XL = (50 – 10) = 40 (iv) XC = (100 – 10) = 90

(v) CD = (500 – 100) = 400 (vi) CM = (1000 – 100) = 900

RULE 4 *When a smaller numeral is placed between two larger numerals, it is always subtracted from the larger numeral immediately following it.*

EXAMPLES (i) XIV = 10 + (5 – 1) = 14 (ii) XIX = 10 + (10 – 1) = 19

(iii) CXIV = 100 + 10 + (5 – 1) = 114

SUMMARY

1.

Roman numeral	I	V	X	L	C	D	M
Hindu-Arabic numeral	1	5	10	50	100	500	1000

2. Repetition of a Roman numeral means addition.

RULES (i) Only I, X, C, M can be repeated.
(ii) V, L and D are never repeated.
(iii) No symbol can be repeated more than 3 times.

3. Smaller numeral written to the left of a larger numeral means subtraction.

RULES (i) V, L and D are never subtracted.
(ii) I can be subtracted from V and X only.
(iii) X can be subtracted from L and C only.
(iv) C can be subtracted from D and M only.

SOLVED EXAMPLES

EXAMPLE 1. *Write Roman numeral for each of the numbers from 1 to 20.*

Solution We may write these numbers as given below.

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

11	12	13	14	15	16	17	18	19	20
XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX

EXAMPLE 2. *Express each of the following numbers as a Roman numeral:*

(i) 23 *(ii)* 26 *(iii)* 29 *(iv)* 31 *(v)* 37 *(vi)* 39 *(vii)* 40
(viii) 45 *(ix)* 49 *(x)* 51 *(xi)* 63 *(xii)* 72 *(xiii)* 79 *(xiv)* 84
(xv) 89 *(xvi)* 90 *(xvii)* 92 *(xviii)* 99 *(xix)* 100

Solution We may write these numbers as given below:

(i) 23 = XXIII (ii) 26 = XXVI (iii) 29 = XXIX
(iv) 31 = XXXI (v) 37 = XXXVII (vi) 39 = XXXIX
(vii) 40 = XL (viii) 45 = XLV (ix) 49 = XLIX
(x) 51 = LI (xi) 63 = LXIII (xii) 72 = LXXII
(xiii) 79 = LXXIX (xiv) 84 = LXXXIV (xv) 89 = LXXXIX
(xvi) 90 = XC (xvii) 92 = XCII (xviii) 99 = XCIX
(xix) 100 = C

EXAMPLE 3. *Express each of the following numbers as a Roman numeral:*

(i) 137 *(ii)* 174 *(iii)* 198 *(iv)* 236
(v) 341 *(vi)* 389 *(vii)* 400 *(viii)* 479
(ix) 556 *(x)* 596 *(xi)* 625 *(xii)* 769

Solution We have:

(i) 137 = 100 + 30 + 7 = CXXXVII (ii) 174 = 100 + 70 + 4 = CLXXIV
(iii) 198 = 100 + 90 + 8 = CXCVIII (iv) 236 = 200 + 30 + 6 = CCXXXVI
(v) 341 = 300 + 40 + 1 = CCCXLI (vi) 389 = 300 + 80 + 9 = CCCLXXXIX
(vii) 400 = CD (viii) 479 = 400 + 70 + 9 = CDLXXIX
(ix) 556 = 500 + 50 + 6 = DLVI (x) 596 = 500 + 90 + 6 = DXCVI
(xi) 625 = 500 + 100 + 20 + 5 = DCXXV (xii) 769 = 500 + 200 + 60 + 9 = DCCLXIX

EXAMPLE 4. *Write each of the following in Hindu-Arabic numeral:*

(i) XXIV *(ii)* XLVI *(iii)* LXXXVI *(iv)* XCIX

(v) CLXVI *(vi)* CCXXVI *(vii)* CCCXL *(viii)* CDXLVI

Solution We have:

(i) XXIV = 20 + (5 − 1) = 24

(ii) XLVI = XL + VI = (50 − 10) + (5 + 1) = 46

(iii) LXXXVI = 50 + 30 + 6 = 86

(iv) XCIX = (100 − 10) + (10 − 1) = 99

(v) CLXVI = 100 + 50 + 10 + (5 + 1) = 166

(vi) CCXXVI = 200 + 20 + (5 + 1) = 226

(vii) CCCXL = 300 + (50 − 10) = 340

(viii) CDXLVI = (500 − 100) + (50 − 10) + (5 + 1) = 446

EXAMPLE 5. *Show that each of the following is meaningless. Give reason in each case.*

(i) XXXX *(ii)* VX *(iii)* IC *(iv)* XVV

Solution (i) No symbol is repeated more than three times.
∴ XXXX is wrong.

(ii) V, L, D are never subtracted.
∴ VX is wrong.

(iii) I can be subtracted from V and X only.
∴ IC is wrong.

(iv) V, L, D are never repeated.
∴ XVV is wrong.

EXERCISE 1G

1. Express each of the following as a Roman numeral:

(i) 2 (ii) 8 (iii) 14 (iv) 29

(v) 36 (vi) 43 (vii) 54 (viii) 61

(ix) 73 (x) 81 (xi) 91 (xii) 95

(xiii) 99 (xiv) 105 (xv) 114

2. Express each of the following as a Roman numeral:

(i) 164 (ii) 195 (iii) 226 (iv) 341

(v) 475 (vi) 596 (vii) 611 (viii) 759

3. Write each of the following as a Hindu-Arabic numeral:

(i) XXVII (ii) XXXIV (iii) XLV (iv) LIV

(v) LXXIV (vi) XCI (vii) XCVI (viii) CXI

(ix) CLIV (x) CCXXIV (xi) CCCLXV (xii) CDXIV

(xiii) CDLXIV (xiv) DVI (xv) DCCLXVI

4. Show that each of the following is meaningless. Give reason in each case.

(i) VC (ii) IL (iii) VVII (iv) IXX

Hint. *(i) V is never subtracted.*

(ii) I can be subtracted from V and X only.

(iii) V, L, D are never repeated.

(iv) IX cannot occur to the left of X.

EXERCISE 1H

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. The place value of 6 in the numeral 48632950 is
   (a) 6 (b) 632950 (c) 600000 (d) 486
2. The face value of 4 in the numeral 89247605 is
   (a) 4 (b) 40000 (c) 47605 (d) 8924
3. The difference between the place value and the face value of 5 in the numeral 78653421 is
   (a) 53416 (b) 4995 (c) 49995 (d) none of these
4. The smallest counting number is
   (a) 0 (b) 1 (c) 10 (d) none of these
5. How many 4-digit numbers are there?
   (a) 8999 (b) 9000 (c) 8000 (d) none of these
6. How many 7-digit numbers are there?
   (a) 8999999 (b) 9000000 (c) 1000000 (d) none of these
7. How many 8-digit numbers are there?
   (a) 99999999 (b) 89999999 (c) 90000000 (d) none of these
8. What comes just before 1000000?
   (a) 99999 (b) 999999 (c) 9999999 (d) none of these
9. Which of the following is not meaningful?
   (a) VX (b) XV (c) XXV (d) XXXV
10. Which of the following is not meaningful?
    (a) CI (b) CII (c) IC (d) XC
11. Which of the following is not meaningful?
    (a) XIV (b) XVV (c) XIII (d) XXII

TEST PAPER-1

A. **1.** Write each of the following numerals in words:
(i) 16, 06, 23, 708 (ii) 14, 23, 08, 915

2. Write each of the following numerals in words:
(i) 80, 060, 409 (ii) 234, 150, 319

3. Arrange the following numbers in ascending order:
3903216, 19430124, 864572, 6940513, 16531079

4. Arrange the following numbers in descending order:
54796203, 4675238, 63240613, 5125648, 589623

5. How many 7-digit numbers are there in all?

6. Write the largest and smallest numbers using each of the digits 1, 4, 6, 8, 0 only once and find their difference.

7. Write the Hindu-Arabic numeral for each of the following:
(i) CCXLII (ii) CDLXV (iii) LXXVI
(iv) DCCXLI (v) XCIV (vi) CXCIX

8. Write the Roman numeral for each of the following:
(i) 84 (ii) 99 (iii) 145 (iv) 406 (v) 519

9. Write the successor and predecessor of 999999 and find their difference.

10. Round off each of the following to the nearest thousand:
(i) 1046 (ii) 973 (iii) 5624 (iv) 4368

B. ***Mark (✓) against the correct answer in each of the following:***

11. Which of the following Roman numerals is correct?
(a) XC (b) XD (c) DM (d) VL

12. 1 Lakh = ... thousands.
(a) 10 (b) 100 (c) 1000 (d) none of these

13. No Roman numeral can be repeated more than ..... times.
(a) two (b) three (c) four (d) none of these

14. How many times does the digit 9 occur between 1 and 100?
(a) 11 (b) 15 (c) 18 (d) 20

15. (7268 – 2427) estimated to the nearest hundred is
(a) 4900 (b) 4800 (c) 4841 (d) 5000

16. One million = ...
(a) 1 lakh (b) 10 lakh (c) 100 lakh (d) 1 crore

17. 1512 when rounded off to the nearest hundred is
(a) 1600 (b) 1500 (c) 1510 (d) none of these

18. Which of the symbols are never repeated?
(a) V, X and C (b) V, X and D (c) V, L and D (d) L, K and C

19. Write 86324805 separating periods in Hindu-Arabic system.

C. **20.** ***Fill in the blanks:***
(i) 1 crore = ... lakh
(ii) 1 crore = ... million
(iii) 564 when estimated to the nearest hundred is ..... .
(iv) The smallest 4-digit number with four different digits is ..... .

D. *Write 'T' for true and 'F' for false in each of the following:*

21. The difference in the face value and the place value of 5 in 85419 is 85414.

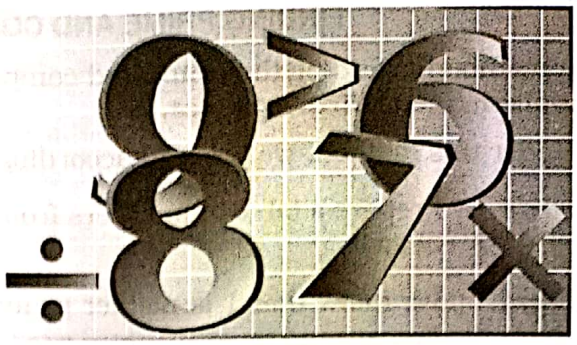
22. In Roman numerals V, L and D are never subtracted.

23. The successor of the greatest 5-digit number is 100000.

24. The estimated value of 46,530 to the nearest hundred is 46500.

25. 100 lakhs make a million.

2 Factors and Multiples



In the previous class we have studied the basic ideas about factors and multiples. In this chapter, we shall review these ideas and extend our study to include some new properties. Here, by numbers we would mean only counting numbers. Recall the following two definitions.

FACTOR *A factor of a number is an exact divisor of that number.*

MULTIPLE *A number is said to be a multiple of any of its factors.*

EXAMPLE We know that $15 = 1 \times 15$ and $15 = 3 \times 5$.
This shows that each of the numbers 1, 3, 5, 15 exactly divides 15.
Therefore, 1, 3, 5, 15 are all *factors* of 15.
In other words, we can say that 15 is a *multiple* of each one of the numbers 1, 3, 5 and 15.

Thus, we conclude that

if a number x divides a number y exactly then x is called a factor of y, and y is called a multiple of x.

Clearly, 1 *is a factor of every number.*

And, *every number is a factor of itself.*

It may be noted that 1 *is the only number which has exactly one factor, namely, itself.*

VARIOUS TYPES OF NUMBERS

(i) EVEN NUMBERS *All multiples of 2 are called even numbers.*

For example: 2, 4, 6, 8, 10, 12, etc., are all even numbers.

(ii) ODD NUMBERS *Numbers which are not multiples of 2 are called odd numbers.*

For example: 1, 3, 5, 7, 9, 11, 13, etc., are all odd numbers.

(iii) PRIME NUMBERS *Each of the numbers which have exactly two factors, namely, 1 and itself, is called a prime number.*

For example: The numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, etc., are all prime numbers.

(iv) COMPOSITE NUMBERS *Numbers having more than two factors are known as composite numbers.*

For example: Each of the numbers 4, 6, 8, 9, 10, 12, 14, etc., is a composite number.

IMPORTANT FACTS

(i) 1 *is neither prime nor composite.*
(ii) 2 *is the lowest prime number.*
(iii) 2 *is the only even prime number.* All other even numbers are composite numbers.

FINDING PRIME AND COMPOSITE NUMBERS FROM 1 to 100

A method for finding the prime and composite numbers from 1 to 100 was found by the Greek mathematician ***Eratosthenes.***

Under this method, we proceed according to the steps given below.

Step 1. Prepare a table of numbers from 1 to 100, taking ten numbers in each row, as shown below.

Step 2. We know that 1 is neither prime nor composite. So, we separate it out by making a box around it.

Step 3. Encircle ② as a prime number and cross out every multiple of 2.

Step 4. Encircle ③ as a prime number and cross out every multiple of 3. We need not mark the numbers which have already been crossed out.

Step 5. Encircle ⑤ as a prime number and cross out every multiple of 5. We need not mark the numbers which have already been crossed out.

Step 6. Continue this process till the numbers up to 100 are either encircled or crossed-out.

SIEVE OF ERATOSTHENES

1	②	③	~~4~~	⑤	~~6~~	⑦	~~8~~	~~9~~	~~10~~
⑪	~~12~~	⑬	~~14~~	~~15~~	~~16~~	⑰	~~18~~	⑲	~~20~~
~~21~~	~~22~~	㉓	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	㉙	~~30~~
㉛	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	㊲	~~38~~	~~39~~	~~40~~
㊶	~~42~~	㊸	~~44~~	~~45~~	~~46~~	㊼	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	㊸	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	㊾	~~60~~
㊶	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	㊼	~~68~~	~~69~~	~~70~~
㊶	~~72~~	㊸	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	㊾	~~80~~
~~81~~	~~82~~	㊸	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	㊾	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	㊼	~~98~~	~~99~~	~~100~~

Note that: (i) 1 is neither prime nor composite.
(ii) All encircled numbers are prime numbers.
(iii) All crossed out numbers are composite numbers.

Thus, all prime numbers from 1 to 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

TWIN PRIMES *Two consecutive odd prime numbers are known as twin primes.*

Pairs of twin primes between 1 and 100 are:

(i) 3, 5 (ii) 5, 7 (iii) 11, 13 (iv) 17, 19
(v) 29, 31 (vi) 41, 43 (vii) 59, 61 (viii) 71, 73

PRIME TRIPLET *A set of three consecutive prime numbers, differing by 2, is called a prime triplet.*

The only prime triplet is (3, 5, 7).

CO-PRIMES *Two numbers are said to be co-primes if they do not have a common factor other than 1.*

EXAMPLES (i) 2, 3 (ii) 3, 4 (iii) 4, 5 (iv) 4, 9 (v) 8, 15

REMARK 1 Two prime numbers are always co-primes.

REMARK 2 Two co-primes need not be prime numbers.

EXAMPLES: 6, 7 are co-primes, while 6 is not a prime number.
9, 10 are co-primes, while none of 9 and 10 is a prime number.

PERFECT NUMBERS *If the sum of all the factors of a number is two times the number, then the number is called a perfect number.*

EXAMPLES (i) 6 *is a perfect number,* since the factors of 6 are 1, 2, 3, 6 and $(1 + 2 + 3 + 6) = (2 \times 6)$.
(ii) 28 *is a perfect number,* since the factors of 28 are 1, 2, 4, 7, 14, 28 and $(1 + 2 + 4 + 7 + 14 + 28) = (2 \times 28)$.

EXERCISE 2A

1. Define: (i) factor (ii) multiple. Give five examples of each.
2. Write down all the factors of
   (i) 20 (ii) 36 (iii) 60 (iv) 75
3. Write the first five multiples of each of the following numbers:
   (i) 17 (ii) 23 (iii) 65 (iv) 70
4. Which of the following numbers are even and which are odd?
   (i) 32 (ii) 37 (iii) 50 (iv) 58
   (v) 69 (vi) 144 (vii) 321 (viii) 253
5. What are prime numbers? Give ten examples.
6. Write all the prime numbers between
   (i) 10 and 40 (ii) 80 and 100 (iii) 40 and 80 (iv) 30 and 40
7. (i) Write the smallest prime number.
   (ii) List all even prime numbers.
   (iii) Write the smallest odd prime number.
8. Find which of the following numbers are primes:
   (i) 87 (ii) 89 (iii) 63 (iv) 91
9. Make a list of seven consecutive numbers, none of which is prime.
   Hint. *See the sieve of Eratosthenes.*
10. (i) Is there any counting number having no factor at all?
    (ii) Find all the numbers having exactly one factor.
    (iii) Find numbers between 1 and 100 having exactly three factors.
11. What are composite numbers? Can a composite number be odd? If yes, write the smallest odd composite number.
12. What are twin primes? Write all the pairs of twin primes between 50 and 100.
13. What are co-primes? Give examples of five pairs of co-primes. Are co-primes always primes? If no, illustrate your answer by an example.
14. Express each of the following numbers as the sum of two odd primes:
    (i) 36 (ii) 42 (iii) 84 (iv) 98
15. Express each of the following odd numbers as the sum of three odd prime numbers:
    (i) 31 (ii) 35 (iii) 49 (iv) 63
16. Express each of the following numbers as the sum of twin primes:

(i) 36 (ii) 84 (iii) 120 (iv) 144

17. Which of the following statements are true?

(i) 1 is the smallest prime number.

(ii) If a number is prime, it must be odd.

(iii) The sum of two prime numbers is always a prime number.

(iv) If two numbers are co-primes, at least one of them must be a prime number.

DIVISIBILITY TESTS FOR 2, 3, 4, 5, 6, 7, 8, 9, 10 AND 11

(i) TEST OF DIVISIBILITY BY 2 *A number is divisible by 2 if its ones digit is 0, 2, 4, 6 or 8.*

EXAMPLE 1. Each of the numbers 30, 52, 84, 136, 2108 is divisible by 2.

EXAMPLE 2. None of the numbers 71, 83, 215, 467, 629 is divisible by 2.

(ii) TEST OF DIVISIBILITY BY 3 *A number is divisible by 3 if the sum of its digits is divisible by 3.*

EXAMPLE 1. Consider the number 64275.
Sum of its digits = (6 + 4 + 2 + 7 + 5) = 24, which is divisible by 3.
Therefore, 64275 is divisible by 3.

EXAMPLE 2. Consider the number 39583.
Sum of its digits = (3 + 9 + 5 + 8 + 3) = 28, which is not divisible by 3.
Therefore, 39583 is not divisible by 3.

(iii) TEST OF DIVISIBILITY BY 4 *A number is divisible by 4 if the number formed by its digits in the tens and ones places is divisible by 4.*

EXAMPLE 1. Consider the number 96852.
The number formed by the tens and ones digits is 52, which is divisible by 4.
Therefore, 96852 is divisible by 4.

EXAMPLE 2. Consider the number 61394.
The number formed by the tens and ones digits is 94, which is not divisible by 4.
Therefore, 61394 is not divisible by 4.

(iv) TEST OF DIVISIBILITY BY 5 *A number is divisible by 5 if its ones digit is 0 or 5.*

EXAMPLE 1. Each of the numbers 65, 195, 230, 310 is divisible by 5.

EXAMPLE 2. None of the numbers 71, 83, 94, 106, 327, 148, 279 is divisible by 5.

(v) TEST OF DIVISIBILITY BY 6 *A number is divisible by 6 if it is divisible by each one of 2 and 3.*
Note that 2 and 3 are the prime factors of 6.

EXAMPLE 1. Each of the numbers 18, 42, 60, 114, 1356 is divisible by 6.

EXAMPLE 2. None of the numbers 21, 25, 34, 52 is divisible by 6.

(vi) TEST OF DIVISIBILITY BY 7 *A number is divisible by 7 if the difference between twice the ones digit and the number formed by the other digits is either 0 or a multiple of 7.*

EXAMPLE 1. Consider the number 6804.
Clearly, (680 − 2 × 4) = 672, which is divisible by 7.
Therefore, 6804 is divisible by 7.

EXAMPLE 2. Consider the number 137.
Clearly, $(2 \times 7) - 13 = 1$, which is not divisible by 7.
Therefore, 137 is not divisible by 7.

EXAMPLE 3. Consider the number 1367.
Clearly, $136 - (2 \times 7) = 136 - 14 = 122$, which is not divisible by 7.
Therefore, 1367 is not divisible by 7.

(vii) TEST OF DIVISIBILITY BY 8 *A number is divisible by 8 if the number formed by its digits in hundreds, tens and ones places is divisible by 8.*

EXAMPLE 1. Consider the number 79152.
The number formed by hundreds, tens and ones digits is 152, which is clearly divisible by 8.
Therefore, 79152 is divisible by 8.

EXAMPLE 2. Consider the number 57348.
The number formed by hundreds, tens and ones digits is 348, which is not divisible by 8.
Therefore, 57348 is not divisible by 8.

(viii) TEST OF DIVISIBILITY BY 9 *A number is divisible by 9 if the sum of its digits is divisible by 9.*

EXAMPLE 1. Consider the number 65403.
Sum of its digits = $(6 + 5 + 4 + 0 + 3) = 18$, which is divisible by 9.
Therefore, 65403 is divisible by 9.

EXAMPLE 2. Consider the number 81326.
Sum of its digits = $(8 + 1 + 3 + 2 + 6) = 20$, which is not divisible by 9.
Therefore, 81326 is not divisible by 9.

(ix) TEST OF DIVISIBILITY BY 10 *A number is divisible by 10 if its ones digit is 0.*

EXAMPLE 1. Each of the numbers 30, 160, 690, 720 is divisible by 10.

EXAMPLE 2. None of the numbers 21, 32, 63, 84, etc., is divisible by 10.

(x) TEST OF DIVISIBILITY BY 11 *A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in even places (starting from the ones place) is either 0 or a multiple of 11.*

EXAMPLE 1. Consider the number 90728.
Sum of its digits in odd places = $(8 + 7 + 9) = 24$.
Sum of its digits in even places = $(2 + 0) = 2$.
Difference of the two sums = $(24 - 2) = 22$, which is clearly divisible by 11.
Therefore, 90728 is divisible by 11.

EXAMPLE 2. Consider the number 863423.
Sum of its digits in odd places = $(3 + 4 + 6) = 13$.
Sum of its digits in even places = $(2 + 3 + 8) = 13$.
Difference of these sums = $(13 - 13) = 0$.
Therefore, 863423 is divisible by 11.

EXAMPLE 3. Consider the number 76844.
Sum of its digits in odd places = $(4 + 8 + 7) = 19$.
Sum of its digits in even places = $(4 + 6) = 10$.
Difference of these sums = $(19 - 10) = 9$, which is not divisible by 11.
Therefore, 76844 is not divisible by 11.

GENERAL PROPERTIES OF DIVISIBILITY

PROPERTY 1. *If a number is divisible by another number, it must be divisible by each of the factors of that number.*

EXAMPLE We know that 36 is divisible by 12.
All factors of 12 are 1, 2, 3, 4, 6, 12.
Clearly, 36 is divisible by each one of 1, 2, 3, 4, 6, 12.

REMARKS As a consequence of the above result, we can say that
(i) every number divisible by 9 is also divisible by 3,
(ii) every number divisible by 8 is also divisible by 4.

PROPERTY 2. *If a number is divisible by each of two co-prime numbers, it must be divisible by their product.*

EXAMPLE 1. We know that 972 is divisible by each of the numbers 2 and 3. Also, 2 and 3 are co-primes.
So, according to Property 2, the number 972 must be divisible by 6, which is true.

EXAMPLE 2. We know that 4320 is divisible by each one of the numbers 5 and 8. Also, 5 and 8 are co-primes.
So, 4320 must be divisible by 40.
By actual division, we find that it is true.

EXAMPLE 3. Consider the number 372.
It may be verified that the above number is divisible by both 4 and 6.
But, by actual division, we find that 372 is not divisible by 24.
Be careful, 4 and 6 are not co-primes.

REMARK Since two prime numbers are always co-primes, it follows that if a number is divisible by each one of any two prime numbers then the number is divisible by their product.

PROPERTY 3. *If a number is a factor of each of the two given numbers, then it must be a factor of their sum.*

EXAMPLE 1. We know that 5 is a factor of 15 as well as that of 20.
So, 5 must be a factor of (15 + 20), that is, 35.
And, this is clearly true.

EXAMPLE 2. We know that 7 is a factor of each of the numbers 49 and 63.
So, 7 must be a factor of (49 + 63) = 112.
Clearly, 7 divides 112 exactly.

PROPERTY 4. *If a number is a factor of each of the two given numbers then it must be a factor of their difference.*

EXAMPLE 1. We know that 3 is a factor of each one of the numbers 36 and 24.
So, 3 must be a factor of (36 – 24) = 12.
Clearly, 3 divides 12 exactly.

EXAMPLE 2. We know that 13 is a factor of each one of the numbers 65 and 117.
So, 13 must be a factor of (117 – 65) = 52.
Clearly, 13 divides 52 exactly.

TO FIND PRIME NUMBERS BETWEEN 100 AND 200

We know that $15 \times 15 > 200$.

So, we adopt the following rule:

Rule *Examine whether the given number is divisible by any prime number less than* 15. *If yes then it is not prime; otherwise it is prime.*

EXAMPLE *Which of the following are prime numbers?*
(i) 117 *(ii)* 139 *(iii)* 193

Solution

(i) Test the divisibility of 117 by each one of the prime numbers 2, 3, 5, 7, 11, 13, taking one by one. We find that 117 is divisible by 13. So, 117 is not a prime number.

(ii) Test the divisibility of 139 by each one of the prime numbers 2, 3, 5, 7, 11, 13. We find that 139 is divisible by none of them. So, 139 is a prime number.

(iii) Test the divisibility of 193 by each one of the prime numbers 2, 3, 5, 7, 11, 13. We find that 193 is divisible by none of them.
So, 193 is a prime number.

TO FIND PRIME NUMBERS BETWEEN 100 AND 400

We know that $20 \times 20 = 400$.

Rule *Examine whether the given number is divisible by any prime number less than* 20. *If yes then it is not prime; otherwise it is prime.*

EXAMPLE *Which of the following is a prime number?*
(i) 263 *(ii)* 323 *(iii)* 361

Solution

(i) Test the divisibility of 263 by each one of the prime numbers 2, 3, 5, 7, 11, 13, 17, 19. We find that 263 is not divisible by any of these numbers.
So, 263 is a prime number.

(ii) Test the divisibility of 323 by each one of the numbers 2, 3, 5, 7, 11, 13, 17, 19. We find that 323 is divisible by 17.
$\therefore$ 323 is not a prime number.

(iii) Test the divisibility of 361 by each one of the prime numbers 2, 3, 5, 7, 11, 13, 17, 19. We find that 361 is divisible by 19.
Hence, 361 is not a prime number.

EXERCISE 2B

1. Test the divisibility of the following numbers by 2:
(i) 2650 (ii) 69435 (iii) 59628
(iv) 789403 (v) 357986 (vi) 367314

2. Test the divisibility of the following numbers by 3:
(i) 733 (ii) 10038 (iii) 20701
(iv) 524781 (v) 79124 (vi) 872645

3. Test the divisibility of the following numbers by 4:
(i) 618 (ii) 2314 (iii) 63712
(iv) 35056 (v) 946126 (vi) 810524

4. Test the divisibility of the following numbers by 5:
(i) 4965 (ii) 23590 (iii) 35208
(iv) 723405 (v) 124684 (vi) 438750

5. Test the divisibility of the following numbers by 6:
(i) 2070 (ii) 46523 (iii) 71232
(iv) 934706 (v) 251780 (vi) 872536

6. Test the divisibility of the following numbers by 7:
(i) 826 (ii) 117 (iii) 2345
(iv) 6021 (v) 14126 (vi) 25368

7. Test the divisibility of the following numbers by 8:
(i) 9364 (ii) 2138 (iii) 36792
(iv) 901674 (v) 136976 (vi) 1790184

8. Test the divisibility of the following numbers by 9:
(i) 2358 (ii) 3333 (iii) 98712
(iv) 257106 (v) 647514 (vi) 326999

9. Test the divisibility of the following numbers by 10:
(i) 5790 (ii) 63215 (iii) 55555

10. Test the divisibility of the following numbers by 11:
(i) 4334 (ii) 83721 (iii) 66311
(iv) 137269 (v) 901351 (vi) 8790322

11. In each of the following numbers, replace * by the smallest number to make it divisible by 3:
(i) 27*4 (ii) 53*46 (iii) 8*711
(iv) 62*35 (v) 234*17 (vi) 6*1054

12. In each of the following numbers, replace * by the smallest number to make it divisible by 9:
(i) 65*5 (ii) 2*135 (iii) 6702*
(iv) 91*67 (v) 6678*1 (vi) 835*86

13. In each of the following numbers, replace * by the smallest number to make it divisible by 11:
(i) 26*5 (ii) 39*43 (iii) 86*72
(iv) 467*91 (v) 1723*4 (vi) 9*8071

14. Test the divisibility of:
(i) 10000001 by 11 (ii) 19083625 by 11 (iii) 2134563 by 9
(iv) 10001001 by 3 (v) 10203574 by 4 (vi) 12030624 by 8

15. Which of the following are prime numbers?
(i) 103 (ii) 137 (iii) 161 (iv) 179
(v) 217 (vi) 277 (vii) 331 (viii) 397

16. Give an example of a number
(i) which is divisible by 2 but not by 4.
(ii) which is divisible by 4 but not by 8.
(iii) which is divisible by both 2 and 8 but not by 16.
(iv) which is divisible by both 3 and 6 but not by 18.

17. Write (T) for true and (F) for false against each of the following statements:
(i) If a number is divisible by 4, it must be divisible by 8.
(ii) If a number is divisible by 8, it must be divisible by 4.
(iii) If a number divides the sum of two numbers exactly, it must exactly divide the numbers separately.
(iv) If a number is divisible by both 9 and 10, it must be divisible by 90.
Hint. *9 and 10 are co-primes.*
(v) A number is divisible by 18 if it is divisible by both 3 and 6.
Hint. *3 and 6 are not co-primes. Consider* 186.

(vi) If a number is divisible by 3 and 7, it must be divisible by 21.
(vii) The sum of two consecutive odd numbers is always divisible by 4.
(viii) If a number divides two numbers exactly, it must divide their sum exactly.

PRIME FACTORIZATION

PRIME FACTOR *A factor of a given number is called a prime factor if this factor is a prime number.*

EXAMPLE 2 and 3 are prime factors of 12.

PRIME FACTORIZATION *To express a given number as a product of prime factors is called prime factorization or complete factorization of the given number.*

EXAMPLE Let us factorize 36 in three different ways as given below:

$36 = 2 \times 18$, $18 = 2 \times 9$, $9 = 3 \times 3$

$36 = 3 \times 12$, $12 = 2 \times 6$, $6 = 2 \times 3$

$36 = 9 \times 4$, $9 = 3 \times 3$, $4 = 2 \times 2$

Thus, $36 = 2 \times 2 \times 3 \times 3$; $36 = 3 \times 2 \times 2 \times 3$; $36 = 3 \times 3 \times 2 \times 2$.

We notice here that in each of the prime factorizations, the factors may be arranged differently but, in fact, they are the same.

Thus, we generalise this result as under.

Every composite number can be factorized into primes in only one way, except for the order of primes.

This property is known as *unique factorization property*.

CONCEPT OF POWER

We write, $2 \times 2 = 2^2$ (read as 2 raised to the power 2),

$2 \times 2 \times 2 = 2^3$ (read as 2 raised to the power 3),

$2 \times 2 \times 2 \times 2 = 2^4$ (read as 2 raised to the power 4),

and so on.

Similarly, $3 \times 3 = 3^2, 3 \times 3 \times 3 = 3^3, 3 \times 3 \times 3 \times 3 = 3^4$, and so on.

In general, $a \times a \times \ldots$ taken m times $= a^m$.

SOLVED EXAMPLES

EXAMPLE 1. *Give the prime factorization of* 1260.

Solution We use the division method, as shown below.

2	1260
2	630
3	315
3	105
5	35
7	7
	1

$\therefore\ 1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^2 \times 3^2 \times 5 \times 7.$

EXAMPLE 2. *Give the prime factorization of* 20570.

Solution We have:

2	20570
5	10285
11	2057
11	187
17	17
	1

Therefore, $20570 = 2 \times 5 \times 11 \times 11 \times 17 = 2 \times 5 \times 11^2 \times 17$.

EXERCISE 2C

Give the prime factorization of each of the following numbers:

1. 12	**2.** 18	**3.** 48	**4.** 56	**5.** 90
6. 136	**7.** 252	**8.** 420	**9.** 637	**10.** 945
11. 1224	**12.** 1323	**13.** 8712	**14.** 9317	**15.** 1035
16. 1197	**17.** 4641	**18.** 4335	**19.** 2907	**20.** 13915

HCF AND LCM

HIGHEST COMMON FACTOR (HCF) *The greatest number which is a common factor of two or more given numbers, is called their* ***highest common factor*** *or* ***greatest common divisor*** *or* ***greatest common measure,*** *written as HCF or GCD or GCM.*

EXAMPLE *Let us find the HCF of* 24 *and* 32.

Solution All the factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

All the factors of 32 are: 1, 2, 4, 8, 16, 32

Common factors of 24 and 32 are: 1, 2, 4, 8

Thus, the highest common factor of 24 and 32 is 8.

Hence, HCF of 24 and 32 = 8.

TO FIND HCF (BY PRIME FACTORIZATION METHOD) *We first find the prime factorization of each of the given numbers. Then, the product of all common prime factors, using the least power of each common prime factor, is the HCF of the given numbers.*

SOLVED EXAMPLES

EXAMPLE 1. *Find the HCF of* 144 *and* 198 *by the prime factorization method.*

Solution We have:

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	198
3	99
3	33
11	11
	1

$\therefore \quad 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2.$

And, $198 = 2 \times 3 \times 3 \times 11 = 2 \times 3^2 \times 11.$

$\therefore$ HCF of 144 and $198 = 2 \times 3^2 = 18.$

EXAMPLE 2. *Find the HCF of 396 and 1080 by the prime factorization method.*

Solution We have:

2	396
2	198
3	99
3	33
11	11
	1

2	1080
2	540
2	270
5	135
3	27
3	9
3	3
	1

So, $396 = 2^2 \times 3^2 \times 11.$

And, $1080 = 2^3 \times 3^3 \times 5.$

Hence, the HCF of 396 and 1080 is $2^2 \times 3^2 = 36.$

EXAMPLE 3. *Find the HCF of 144, 180 and 192 by the prime factorization method.*

Solution We have:

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	180
2	90
3	45
3	15
5	5
	1

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

So, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2;$

$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5;$

$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^6 \times 3.$

Therefore, the HCF of the given numbers $= 2^2 \times 3 = 12.$

TO FIND HCF (BY DIVISION METHOD) *Suppose two numbers are given. Divide the greater number by the smaller one. Next, divide the divisor by the remainder. Go on repeating the process of dividing the preceding divisor by the remainder last obtained till the remainder zero is obtained. Then the last divisor is the required HCF of the given numbers.*

EXAMPLE 4. *Find the HCF of 161 and 345 by the division method.*

Solution We have:

$$161\overline{)345}(2$$
$$-322$$
$$23\overline{)161}(7$$
$$-161$$
$$\times$$

Hence, the HCF of 161 and 345 is 23.

EXAMPLE 5. *Find the HCF of 513 and 783.*

Solution By the division method, we have:

$$513\overline{)783}\,(1$$
$$-513$$
$$270\overline{)513}\,(1$$
$$-270$$
$$243\overline{)270}\,(1$$
$$-243$$
$$27\overline{)243}\,(9$$
$$-243$$
$$\times$$

Hence, the HCF of 513 and 783 is 27.

TO FIND THE HCF OF MORE THAN TWO NUMBERS *If more than two numbers are given, choose any two of them and find their HCF. The HCF of this HCF and the third number gives the HCF of these three numbers. The HCF of this HCF and the fourth number gives the HCF of these four numbers, and so on.*

EXAMPLE 6. *Find the HCF of 136, 170 and 255.*

Solution First we find the HCF of 136 and 170.

$$136\overline{)170}\,(1$$
$$-136$$
$$34\overline{)136}\,(4$$
$$-136$$
$$\times$$

Thus, the HCF of 136 and 170 is 34.
Now, we find the HCF of 34 and 255.

$$34\overline{)255}\,(7$$
$$-238$$
$$17\overline{)34}\,(2$$
$$-34$$
$$\times$$

So, the HCF of 34 and 255 is 17.
Hence, the HCF of 136, 170 and 255 is 17.

EXAMPLE 7. *Find the greatest number which divides 285 and 1249, leaving remainders 9 and 7 respectively.*

Solution Clearly, we must find the greatest number which divides (285 – 9) and (1249 – 7) exactly.

So, the required number = HCF of 276 and 1242.

$$276\overline{)1242}\,(4$$
$$-1104$$
$$138\overline{)276}\,(2$$
$$-276$$
$$\times$$

Hence, the required number = 138.

EXAMPLE 8. *Reduce* $\frac{289}{391}$ *to the lowest terms.*

Solution For reducing the given fraction to the lowest terms, we divide its numerator and the denominator by their HCF.
Now, we find the HCF of 289 and 391 as under:

$$\begin{array}{r} 289\overline{)391}(1 \\ -289 \\ \hline 102\overline{)289}(2 \\ -204 \\ \hline 85\overline{)102}(1 \\ -85 \\ \hline 17\overline{)85}(5 \\ -85 \\ \hline \times \end{array}$$

Hence, the HCF of 289 and 391 is 17.
Now, dividing the numerator and the denominator of the given fraction by 17, we get:

$$\frac{289}{391} = \frac{289 \div 17}{391 \div 17} = \frac{17}{23}.$$

EXAMPLE 9. *The length, breadth and height of a room are* 1050 cm, 750 cm *and* 425 cm *respectively. Find the length of the longest tape which can measure the three dimensions of the room exactly.*

Solution The length of the longest tape which can measure the given lengths
= HCF of 1050 cm, 750 cm and 425 cm.
First we find the HCF of 1050 and 750.

$$\begin{array}{r} 750\overline{)1050}(1 \\ -750 \\ \hline 300\overline{)750}(2 \\ -600 \\ \hline 150\overline{)300}(2 \\ -300 \\ \hline \times \end{array}$$

Hence, HCF of 1050 and 750 = 150.
Now, we find the HCF of 150 and 425.

$$\begin{array}{r} 150\overline{)425}(2 \\ -300 \\ \hline 125\overline{)150}(1 \\ -125 \\ \hline 25\overline{)125}(5 \\ -125 \\ \hline \times \end{array}$$

Hence, HCF of 150 and 425 is 25.
Thus, the HCF of 1050, 750 and 425 is 25.
Hence, the required length = 25 cm.

EXERCISE 2D

Find the HCF of the numbers in each of the following, using the prime factorization method:

1. 84, 98
2. 170, 238
3. 504, 980
4. 72, 108, 180
5. 84, 120, 138
6. 106, 159, 371
7. 272, 425
8. 144, 252, 630
9. 1197, 5320, 4389

Find the HCF of the numbers in each of the following, using the division method:

10. 58, 70
11. 399, 437
12. 1045, 1520
13. 1965, 2096
14. 2241, 2324
15. 658, 940, 1128
16. 754, 1508, 1972
17. 391, 425, 527
18. 1794, 2346, 4761

Show that the following pairs are co-primes:

19. 59, 97
20. 161, 192
21. 343, 432
22. 512, 945
23. 385, 621
24. 847, 1014

Hint. *Two numbers are co-primes if their HCF is* 1.

25. Find the greatest number which divides 615 and 963, leaving the remainder 6 in each case.
26. Find the greatest number which divides 2011 and 2623, leaving remainders 9 and 5 respectively.
27. Find the greatest number that will divide 445, 572 and 699, leaving remainders 4, 5, 6 respectively.
28. Reduce each of the following fractions to the lowest terms:
    (i) $\frac{161}{207}$ (ii) $\frac{517}{799}$ (iii) $\frac{296}{481}$
29. Three pieces of timber, 42-m, 49-m and 63-m long, have to be divided into planks of the same length. What is the greatest possible length of each plank?
30. Three different containers contain 403 L, 434 L and 465 L of milk respectively. Find the capacity of a container which can measure the milk of all the containers in an exact number of times.
31. There are 527 apples, 646 pears and 748 oranges. These are to be arranged in heaps containing the same number of fruits. Find the greatest number of fruits possible in each heap. How many heaps are formed?
32. Determine the longest tape which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm.

    Hint. *Convert all the lengths to* cm *and then take the HCF.*
33. A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.
34. Find the HCF of
    (i) two prime numbers (ii) two consecutive numbers
    (iii) two co-primes (iv) 2 and an even number

LOWEST COMMON MULTIPLE (LCM) *The lowest common multiple of two or more numbers is the smallest number which is a multiple of each of the numbers.*

EXAMPLE *Let us find the LCM of* 4 *and* 6.

Solution Multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

Multiples of 6 are: 6, 12, 18, 24, 30, 36, ...

Common multiples of 4 and 6 are: 12, 24, 36, ...

Lowest common multiple of 4 and 6 is 12.

Hence, LCM of 4 and 6 = 12.

TO FIND LCM (BY PRIME FACTORIZATION METHOD) *In order to find the LCM of two or more given numbers we write the prime factorization of each of the given numbers. Then, the required LCM of these numbers is the product of all different prime factors of the numbers, using the greatest power of each common prime factor.*

SOLVED EXAMPLES

EXAMPLE 1. *Find the LCM of 24, 36 and 40 by the prime factorization method.*

Solution We have:

2 | 24
2 | 12
2 | 6
3 | 3
| 1

2 | 36
2 | 18
3 | 9
3 | 3
| 1

2 | 40
2 | 20
2 | 10
5 | 5
| 1

$\therefore \quad 24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

$40 = 2^3 \times 5$

Hence, the LCM of 24, 36 and 40 is $2^3 \times 3^2 \times 5 = 360$.

EXAMPLE 2. *Find the LCM of 112, 168, 266 by the prime factorization method.*

Solution We have:

2 | 112
2 | 56
2 | 28
2 | 14
7 | 7
| 1

2 | 168
2 | 84
2 | 42
3 | 21
7 | 7
| 1

2 | 266
7 | 133
19 | 19
| 1

$\therefore \quad 112 = 2^4 \times 7$

$168 = 2^3 \times 3 \times 7$

$266 = 2 \times 7 \times 19$

Therefore, the LCM of the given numbers = $2^4 \times 3 \times 7 \times 19 = 6384$.

TO FIND LCM (BY DIVISION METHOD) *In this method, we arrange the given numbers in a line, in any order. We divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. This process is repeated till no two of the given numbers are divisible by a common number. The product of the divisors and the undivided numbers is the required LCM of the given numbers.*

EXAMPLE 3. *Find the LCM of 12, 15, 20, 27 by the division method.*

Solution We have:

3 | 12, 15, 20, 27
4 | 4, 5, 20, 9
5 | 1, 5, 5, 9
| 1, 1, 1, 9

Hence, the LCM of the given numbers = $3 \times 4 \times 5 \times 9 = 540$.

EXAMPLE 4. *Find the LCM of* 22, 54, 108, 135 *and* 198.

Solution We have:

2	22, 54, 108, 135, 198
11	11, 27, 54, 135, 99
9	1, 27, 54, 135, 9
3	1, 3, 6, 15, 1
	1, 1, 2, 5, 1

Hence, the LCM of the given numbers $= 2 \times 11 \times 9 \times 3 \times 2 \times 5 = 5940$.

EXAMPLE 5. *Find the smallest number which when diminished by* 3 *is divisible by* 21, 28, 36 *and* 45.

Solution We know that the smallest number divisible by 21, 28, 36 and 45 is their LCM. We calculate this LCM as under:

7	21, 28, 36, 45
3	3, 4, 36, 45
3	1, 4, 12, 15
4	1, 4, 4, 5
	1, 1, 1, 5

Hence, the LCM of 21, 28, 36 and 45 is $7 \times 3 \times 3 \times 4 \times 5 = 1260$.

Hence, the required number $= (1260 + 3) = 1263$.

EXAMPLE 6. *In a shop, there are three clocks which chime at intervals of* 15, 20 *and* 30 *minutes respectively. They all chime together at* 10 *a.m. At what time will they all chime together again?*

Solution Required time = LCM of 15, 20, 30 minutes.

5	15, 20, 30
3	3, 4, 6
2	1, 4, 2
	1, 2, 1

$\therefore$ LCM of 15, 20, 30 $= (5 \times 3 \times 2 \times 2) = 60$.

So, all the clocks will chime together again after 60 minutes, i.e., after 1 hour, i.e., at 11 a.m.

PROPERTIES OF HCF AND LCM OF GIVEN NUMBERS

(i) *The HCF of a group of numbers is not greater than any of the given numbers.*

(ii) *The HCF of two co-primes is* 1.

(iii) *The LCM of a group of numbers is not less than any of the given numbers.*

(iv) *The LCM of two co-primes is equal to their product.*

(v) *The HCF of a group of numbers is always a factor of their LCM.*

EXAMPLE Consider the numbers 12, 16, 36, 40.

Clearly, the HCF of the given numbers = 4.

And, their LCM

$= 2 \times 2 \times 3 \times 2 \times 2 \times 3 \times 5 = 720$.

2	12, 16, 36, 40
2	6, 8, 18, 20
3	3, 4, 9, 10
2	1, 4, 3, 10
	1, 2, 3, 5

Clearly, 4 is a factor of 720.

(vi) *If a and b are two given numbers such that a is a factor of b then their HCF = a and their LCM = b.*

EXAMPLE We know that 8 is a factor of 32.

Then, clearly HCF of 8 and 32 is 8.

And, LCM of 8 and 32 is 32.

(vii) **AN IMPORTANT PROPERTY** *If two numbers are given then the product of the two numbers = the product of their HCF and LCM.*

EXAMPLE Consider the numbers 48 and 60.

We have, $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$.

And, $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$.

So, the HCF of 48 and 60 is $2^2 \times 3 = 12$.

And, the LCM of 48 and 60 is $2^4 \times 3 \times 5 = 240$.

Now, the product of the given numbers = $48 \times 60 = 2880$.

Product of their HCF and LCM = $(12 \times 240) = 2880$.

$\therefore$ *product of two numbers = (their HCF) × (their LCM).*

REMARKS Thus, for any two given numbers, we have:

(i) $\text{LCM} = \dfrac{\text{(one number)} \times \text{(the other number)}}{\text{their HCF}}$

(ii) $\text{HCF} = \dfrac{\text{(one number)} \times \text{(the other number)}}{\text{their LCM}}$

EXAMPLE 7. *Find the HCF and the LCM of* 1152 *and* 1664.

Solution We first find the HCF of the given numbers.

$$
\begin{array}{r}
1152\overline{)1664}(1 \\
-1152 \\
\hline
512\overline{)1152}(2 \\
-1024 \\
\hline
128\overline{)512}(4 \\
-512 \\
\hline
\times
\end{array}
$$

$\therefore$ HCF = 128.

And, $\text{LCM} = \dfrac{\text{product of the numbers}}{\text{their HCF}} = \dfrac{1152 \times 1664}{128} = 14976.$

$\therefore$ HCF = 128 and LCM = 14976.

EXAMPLE 8. *The HCF of two numbers is* 16 *and their product is* 3072. *Find their LCM.*

Solution We know that

$$\text{LCM} = \frac{\text{product of the given two numbers}}{\text{their HCF}} = \frac{3072}{16} = 192.$$

EXAMPLE 9. *The HCF of two numbers is* 23 *and their LCM is* 1449. *If one of the numbers is* 161, *find the other.*

Solution We know that

(one number) × (the other number) = (HCF × LCM).

Hence, the required number $= \left(\frac{23 \times 1449}{161}\right) = 207.$

EXAMPLE10. *Can two numbers have* 16 *as their HCF and* 204 *as their LCM? Give reason.*

Solution We know that the HCF of two or more numbers must divide their LCM exactly.

But, 16 does not divide 204 exactly.

So, there can be no two numbers with 16 as their HCF and 204 as their LCM.

EXERCISE 2E

Find the LCM of the numbers given below:

1. 42, 63 **2.** 60, 75 **3.** 12, 18, 20 **4.** 36, 60, 72

5. 36, 40, 126 **6.** 16, 28, 40, 77 **7.** 28, 36, 45, 60 **8.** 144, 180, 384

9. 48, 64, 72, 96, 108

Find the HCF and LCM of

10. 117, 221 **11.** 234, 572 **12.** 693, 1078 **13.** 145, 232

14. 861, 1353 **15.** 2923, 3239

16. For each pair of numbers, verify that their product = (HCF × LCM).

(i) 87, 145 (ii) 186, 403 (iii) 490, 1155

17. The product of two numbers is 2160 and their HCF is 12. Find their LCM.

18. The product of two numbers is 2560 and their LCM is 320. Find their HCF.

19. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, find the other.

20. The HCF and LCM of two numbers are 131 and 8253 respectively. If one of the numbers is 917, find the other.

21. Find the least number divisible by 15, 20, 24, 32 and 36.

22. Find the least number which when divided by 25, 40 and 60 leaves 9 as the remainder in each case.

23. Find the least number of five digits that is exactly divisible by 16, 18, 24 and 30.

24. Find the greatest number of five digits exactly divisible by 9, 12, 15, 18 and 24.

25. Three bells toll at intervals of 9, 12, 15 minutes. If they start tolling together, after what time will they next toll together?

26. Three boys step off together from the same place. If their steps measure 36 cm, 48 cm and 54 cm, at what distance from the starting point will they again step together?

27. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds. If they start changing simultaneously at 8 a.m., after how much time will they change again simultaneously?

28. Three measuring rods are 45 cm, 50 cm and 75 cm in length. What is the least length (in metres) of a rope that can be measured by the full length of each of these three rods?

29. An electronic device makes a beep after every 15 minutes. Another device makes a beep after every 20 minutes. They beeped together at 6 a.m. At what time will they next beep together?

30. The circumferences of four wheels are 50 cm, 60 cm, 75 cm and 100 cm. They start moving simultaneously. What least distance should they cover so that each wheel makes a complete number of revolutions?

EXERCISE 2F

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. Which of the following numbers is divisible by 3?
   (a) 24357806 (b) 35769812 (c) 83479560 (d) 3336433
2. Which of the following numbers is divisible by 9?
   (a) 8576901 (b) 96345210 (c) 67594310 (d) none of these
3. Which of the following numbers is divisible by 4?
   (a) 78653234 (b) 98765042 (c) 24689602 (d) 87941032
4. Which of the following numbers is divisible by 8?
   (a) 96354142 (b) 37450176 (c) 57064214 (d) none of these
5. Which of the following numbers is divisible by 6?
   (a) 8790432 (b) 98671402 (c) 85492014 (d) none of these
6. Which of the following numbers is divisible by 11?
   (a) 3333333 (b) 1111111 (c) 22222222 (d) none of these
7. Which of the following is a prime number?
   (a) 81 (b) 87 (c) 91 (d) 97
8. Which of the following is a prime number?
   (a) 117 (b) 171 (c) 179 (d) none of these
9. Which of the following is a prime number?
   (a) 323 (b) 361 (c) 263 (d) none of these
10. Which of the following are co-primes?
    (a) 8, 12 (b) 9, 10 (c) 6, 8 (d) 15, 18
11. Which of the following is a composite number?
    (a) 23 (b) 29 (c) 32 (d) none of these
12. The HCF of 144 and 198 is
    (a) 9 (b) 12 (c) 6 (d) 18
13. The HCF of 144, 180 and 192 is
    (a) 12 (b) 16 (c) 18 (d) 8
14. Which of the following are co-primes?
    (a) 39, 91 (b) 161, 192 (c) 385, 462 (d) none of these

    Hint. *HCF of co-primes is* 1.

15. $\frac{289}{391}$ when reduced to the lowest terms is
    (a) $\frac{11}{23}$ (b) $\frac{13}{31}$ (c) $\frac{17}{31}$ (d) $\frac{17}{23}$
16. The greatest number which divides 134 and 167 leaving 2 as remainder in each case is
    (a) 14 (b) 17 (c) 19 (d) 33
17. The LCM of 24, 36, 40 is
    (a) 4 (b) 90 (c) 360 (d) 720

18. The LCM of 12, 15, 20, 27 is
(a) 270 (b) 360 (c) 480 (d) 540

19. The smallest number which when diminished by 3 is divisible by 14, 28, 36 and 45, is
(a) 1257 (b) 1260 (c) 1263 (d) none of these

20. The HCF of two co-primes is
(a) the smaller number (b) the larger number
(c) 1 (d) none of these

21. If a and b are co-primes, then their LCM is
(a) 1 (b) $\frac{a}{b}$ (c) ab (d) none of these

22. The product of two numbers is 2160 and their HCF is 12. The LCM of these numbers is
(a) 12 (b) 25920 (c) 180 (d) none of these

23. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, the other number is
(a) 290 (b) 435 (c) 5 (d) none of these

24. The least number divisible by each of the numbers 15, 20, 24,32 and 36 is
(a) 1660 (b) 2880 (c) 1440 (d) none of these

25. Three bells toll together at intervals of 9, 12, 15 minutes. If they start tolling together, after what time will they next toll together?
(a) 1 hour (b) $1\frac{1}{2}$ hours (c) $2\frac{1}{2}$ hours (d) 3 hours

Things to Remember

1. *Suppose a number x divides a number y exactly. Then, we say that x is a factor of y. Also, in this case, we say that y is a multiple of x.*
2. *1 is the only number having exactly one factor.*
3. *A number having exactly two factors is called a prime number.*
4. *The only even prime number is 2.*
5. *A number is divisible*
   (i) *by 2, if its ones digit is any of 0, 2, 4, 6, 8;*
   (ii) *by 3, if the sum of its digits is divisible by 3;*
   (iii) *by 9, if the sum of its digits is divisible by 9;*
   (iv) *by 5, if its ones digit is 0 or 5;*
   (v) *by 6, if it is divisible by both 2 and 3;*
   (vi) *by 4, if the number formed by the tens and ones digits is divisible by 4;*
   (vii) *by 8, if the number formed by the hundreds, tens and ones digits is divisible by 8;*
   (viii) *by 11, if the difference of the sum of the digits at odd places and the sum of the digits at even places (beginning from the ones place) is either 0 or a multiple of 11.*
6. *The HCF of two co-primes is 1.*
7. *If x is a factor of y then the HCF of x and y is x, and the LCM of x and y is y.*
8. *The HCF of two or more than two numbers is a factor of their LCM.*
9. *The product of the HCF and LCM of two numbers is equal to the product of the numbers.*

TEST PAPER-2

A. 1. Test the divisibility of 5869473 by 11.

2. Test the divisibility of 67529124 by 8.

3. On dividing 5035 by 31, the remainder is 13. Find the quotient.

4. The HCF of two number is 15 and their product is 1650. Find their LCM.

5. Find the least 5-digit number which is exactly divisible by 20, 25, 30.

6. Find the largest number which divides 630 and 940 leaving remainders 6 and 4 respectively.

7. Find the least number which when divided by 16, 36 and 40 leaves 5 as remainder in each case.

8. Write all prime numbers between 50 and 100.

9. Write seven consecutive composite numbers less than 100 having no prime number between them.

10. Can two numbers have 12 as their HCF and 512 as their LCM? Justify your answer.

B. ***Mark (✓) against the correct answer in each of the following:***

11. Which of the following are co-primes?
(a) 91 and 72 (b) 34 and 51 (c) 21 and 36 (d) 15 and 20

12. The LCM of two co-prime numbers is their
(a) sum (b) difference (c) product (d) quotient

13. The number which is neither prime nor composite is
(a) 0 (b) 1 (c) 2 (d) 3

14. What least number should be replaced for * so that the number 67301*2 is exactly divisible by 9?
(a) 5 (b) 6 (c) 7 (d) 8

15. Which of the following numbers is divisible by 6?
(a) 67821 (b) 78134 (c) 87432 (d) none of these

16. Which of the following is a prime number?
(a) 143 (b) 131 (c) 147 (d) 161

17. $\frac{289}{391}$ when reduced to lowest term is
(a) $\frac{13}{17}$ (b) $\frac{17}{19}$ (c) $\frac{17}{23}$ (d) $\frac{17}{21}$

18. Every counting number has an infinite number of
(a) factors (b) multiples (c) prime factors (d) none of these

C. 19. ***Fill in the blanks.***

(i) 1 is neither ..... nor ..... .

(ii) The smallest prime number is ..... .

(iii) The smallest composite number is ..... .

(iv) The HCF of two consecutive odd numbers is ..... .

(v) Two perfect numbers are ..... and ..... .

D. 20. ***Write 'T' for true and 'F' for false statement.***

(i) Every prime number is odd.

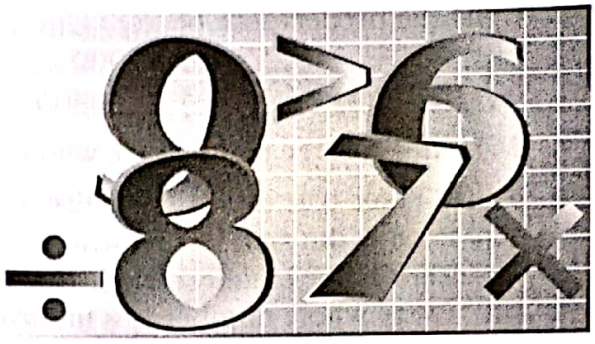
(ii) Every even number is composite.

(iii) The sum of two odd numbers is always odd.

(iv) The sum of two even numbers is always even.

(v) The HCF of two given numbers is always a factor of their LCM.

3 Whole Numbers



NATURAL NUMBERS We are already familiar with the counting numbers 1, 2, 3, 4, 5, 6, etc. *Counting numbers are called natural numbers.*

WHOLE NUMBERS *All natural numbers together with '0' are called whole numbers.*

Thus 0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, ... are whole numbers.

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

SUCCESSOR OF A WHOLE NUMBER *If we add 1 to a whole number, we get the next whole number, called its successor.*

Thus, the successor of 0 is 1, the successor of 1 is 2, the successor of 12 is 13, and so on.

Every whole number has its successor.

PREDECESSOR OF A WHOLE NUMBER *One less than a given whole number (other than 0), is called its predecessor.*

Thus, the predecessor of 1 is 0, the predecessor of 2 is 1, the predecessor of 10 is 9, and so on.

The whole number 0 does not have its predecessor.

Every whole number other than 0 has its predecessor.

EXAMPLE *Write the successor and predecessor of*
(i) 1000 *(ii)* 1005399 *(iii)* 999999

Solution
(i) The successor of 1000 = (1000 + 1) = 1001.
The predecessor of 1000 = (1000 − 1) = 999.
(ii) The successor of 1005399 = (1005399 + 1) = 1005400.
The predecessor of 1005399 = (1005399 − 1) = 1005398.
(iii) The successor of 999999 = (999999 + 1) = 1000000.
The predecessor of 999999 = (999999 − 1) = 999998.

EXERCISE 3A

1. Write the next three whole numbers after 30999.
2. Write the three whole numbers occurring just before 10001.
3. How many whole numbers are there between 1032 and 1209?
4. Which is the smallest whole number?
5. Write the successor of:
(i) 2540801 (ii) 9999 (iii) 50904 (iv) 61639
(v) 687890 (vi) 5386700 (vii) 6475999 (viii) 9999999

6. Write the predecessor of:
   (i) 97 (ii) 10000 (iii) 36900 (iv) 7684320
   (v) 1566391 (vi) 2456800 (vii) 100000 (viii) 1000000
7. Write down three consecutive whole numbers just preceding 7510001.
8. Write (T) for true and (F) for false against each of the following statements:
   (i) Zero is the smallest natural number.
   (ii) Zero is the smallest whole number.
   (iii) Every whole number is a natural number.
   (iv) Every natural number is a whole number.
   (v) 1 is the smallest whole number.
   (vi) The natural number 1 has no predecessor.
   (vii) The whole number 1 has no predecessor.
   (viii) The whole number 0 has no predecessor.
   (ix) The predecessor of a two-digit number is never a single-digit number.
   (x) The successor of a two-digit number is always a two-digit number.
   (xi) 500 is the predecessor of 499.
   (xii) 7000 is the successor of 6999.

OPERATIONS ON WHOLE NUMBERS

We are already familiar with the four basic operations of addition, subtraction, multiplication and division on whole numbers. Now, we shall study the properties of these operations on whole numbers.

PROPERTIES OF ADDITION

(i) CLOSURE PROPERTY *If a and b are any two whole numbers, then $(a+b)$ is also a whole number.*

Let us take some pairs of whole numbers and add them. Check whether the sum is a whole number.

One whole number	Another whole number	Sum	Is the sum a whole number?
9	11	$9+11=20$	Yes
14	28	$14+28=42$	Yes
53	40	$53+40=93$	Yes

Thus, we conclude that the sum of any two whole numbers is a whole number.

(ii) COMMUTATIVE LAW *If a and b are any two whole numbers, then*

$$(a+b)=(b+a).$$

Take a pair of whole numbers. Add them in two different orders and see whether the sum remains the same. Repeat it with more pairs.

EXAMPLES (i) $(8+11=19)$ and $(11+8=19)$

Is $(8+11)=(11+8)$? Yes.

(ii) $(12+23=35)$ and $(23+12=35)$

Is $(12+23)=(23+12)$? Yes.

Thus, we conclude that in whatever order we add two whole numbers, the sum remains the same.

iii) ADDITIVE PROPERTY OF ZERO *If a is any whole number, then*

$$a + 0 = 0 + a = a.$$

XAMPLES We have:

(i) $235 + 0 = 235$ and $0 + 235 = 235$.

(ii) $479 + 0 = 479$ and $0 + 479 = 479$.

iv) ASSOCIATIVE LAW *For any whole numbers a, b, c we always have*

$$(a + b) + c = a + (b + c).$$

EXAMPLE 1. Let us take three whole numbers, say 9, 12 and 15. Then,

$(9 + 12) + 15 = 21 + 15 = 36.$

And, $9 + (12 + 15) = 9 + 27 = 36.$

$\therefore \quad (9 + 12) + 15 = 9 + (12 + 15).$

We may take some more examples and in each case we shall find that in addition of whole numbers, associative law always holds.

REMARK *While adding three or more numbers, we group them in such a way that the calculation becomes easier. We arrange them suitably and add.*

EXAMPLE 2. *Find the sum of* 645, 287 *and* 413.

Solution We have:

$$645 + 287 + 413 = 645 + (287 + 413)$$
$$= (645 + 700) = 1345.$$

EXAMPLE 3. *Find the sum by suitable rearrangement:*

(i) $847 + 306 + 453$ *(ii)* $1852 + 653 + 1648 + 547$

Solution We have:

(i) $847 + 306 + 453 = (847 + 453) + 306$

$= (1300 + 306) = 1606.$

(ii) $1852 + 653 + 1648 + 547 = (1852 + 1648) + (653 + 547)$

$= (3500 + 1200) = 4700.$

EXAMPLE 4. *Find the sum:*

(i) $3678 + 999$ *(ii)* $34876 + 9999$

Solution We have:

(i) $3678 + 999 = 3678 + (1000 - 1)$

$= (3678 + 1000) - 1 = (4678 - 1) = 4677.$

(ii) $34876 + 9999 = 34876 + (10000 - 1)$

$= (34876 + 10000) - 1 = (44876 - 1) = 44875.$

MAGIC SQUARE *A magic square is an arrangement of different numbers in the form of a square such that the sum of the numbers in every horizontal line, every vertical line and every diagonal line is the same.*

One magic square is shown here.

It may be noted that:

Rowwise sum $= (9 + 2 + 7) = (4 + 6 + 8) = (5 + 10 + 3) = 18.$

Columnwise sum $= (9 + 4 + 5) = (2 + 6 + 10) = (7 + 8 + 3) = 18.$

Diagonalwise sum $= (9 + 6 + 3) = (7 + 6 + 5) = 18.$

9	2	7
4	6	8
5	10	3

EXERCISE 3B

1. Fill in the blanks to make each of the following a true statement:
   (i) $458 + 639 = 639 + ...$ (ii) $864 + 2006 = 2006 + ...$
   (iii) $1946 + ... = 984 + 1946$ (iv) $8063 + 0 = ...$
   (v) $53501 + (574 + 799) = 574 + (53501 + ...)$
2. Add the following numbers and check by reversing the order of the addends:
   (i) 16509 + 114 (ii) 2359 + 548 (iii) 19753+2867
3. Find the sum: $(1546 + 498) + 3589$.
   Also, find the sum: $1546 + (498 + 3589)$.
   Are the two sums equal?
   State the property satisfied.
4. Determine each of the sums given below using suitable rearrangement.
   (i) $953 + 707 + 647$ (ii) $1983 + 647 + 217 + 353$
   (iii) $15409 + 278 + 691 + 422$ (iv) $3259 + 10001 + 2641 + 9999$
   (v) $1 + 2 + 3 + 4 + 96 + 97 + 98 + 99$ (vi) $2 + 3 + 4 + 5 + 45 + 46 + 47 + 48$
5. Find the sum by short method:
   (i) 6784 + 9999 (ii) 10578 + 99999
6. For any whole numbers a, b, c, is it true that $(a + b) + c = a + (c + b)$? Give reasons.
7. Complete each one of the following magic squares by supplying the missing numbers:

(i)

	9	2
	5	
8		

(ii)

16	2	
	10	
		4

(iii)

2	15	16	
9	12		
		7	10
14			17

(iv)

	18	17	4
		14	11
	9	10	
19			16

8. Write (T) for true and (F) for false for each of the following statements:
   (i) The sum of two odd numbers is an odd number.
   (ii) The sum of two even numbers is an even number.
   (iii) The sum of an even number and an odd number is an odd number.

SUBTRACTION IN WHOLE NUMBERS

The operation of subtraction is an inverse process of addition.

$(14 + 9 = 23) \Rightarrow \{(23 - 9) = 14 \text{ and } (23 - 14) = 9\}$.

PROPERTIES OF SUBTRACTION

(i) *If a and b are two whole numbers such that $a > b$ or $a = b$ then $a - b$ is a whole number; otherwise, subtraction is not possible in whole numbers.*

EXAMPLES (i) If we subtract two equal whole numbers, we get the whole number 0;
e.g., $(8-8)=0$, $(6-6)=0$, $(25-25)=0$, etc.

(ii) If we subtract a smaller whole number from a larger one, we always get a whole number;
e.g., $(16-9)=7$, $(37-8)=29$, $(23-16)=7$, etc.

(iii) Clearly, we cannot subtract 18 from 13;
i.e., $(13-18)$ is not defined in whole numbers.

(ii) *For any two whole numbers a and b, $(a-b) \neq (b-a)$.*

EXAMPLES (i) $(8-5)=3$ but $(5-8)$ is not defined in whole numbers.
(ii) $(26-9)=17$ but $(9-26)$ is not defined in whole numbers.

(iii) *For any whole number a, we have: $(a-0)=a$ but $(0-a)$ is not defined in whole numbers.*

EXAMPLES (i) $(9-0)=9$ but $(0-9)$ is not defined in whole numbers.
(ii) $(24-0)=24$ but $(0-24)$ is not defined in whole numbers.

(iv) *If a, b, c are any three whole numbers, then in general* $\boldsymbol{(a-b)-c \neq a-(b-c)}$.

EXAMPLE Consider the numbers 8, 4 and 2.
$(8-4)-2=(4-2)=2$.
$8-(4-2)=(8-2)=6$.
$\therefore$ $(8-4)-2 \neq 8-(4-2)$.

(v) *If a, b, c are whole numbers such that $a-b=c$, then $b+c=a$.*

EXAMPLES (i) $16-9=7 \Rightarrow 9+7=16$.
(ii) $23-8=15 \Rightarrow 8+15=23$.

OBSERVING PATTERNS

Study the following:

(i) $456-99=456-100+1=(457-100)=357$.
(ii) $4962-999=4962-1000+1=(4963-1000)=3963$.

EXERCISE 3C

1. Perform the following subtractions. Check your results by the corresponding additions.
(i) 6237 – 694 (ii) 21205 – 10899
(iii) 100000 – 78987 (iv) 1010101 – 656565

2. Replace each * by the correct digit in each of the following:

(i)
$$\begin{array}{r} 9\,1\,7 \\ -\ *\,5\,* \\ \hline 5\,*\,8 \end{array}$$

(ii)
$$\begin{array}{r} 6\,1\,7\,2 \\ -\ *\,*\,6\,9 \\ \hline 2\,9\,*\,* \end{array}$$

(iii)
$$\begin{array}{r} 5\,0\,0\,1\,0\,0\,3 \\ -\ \ \ *\,*\,6\,9\,8\,7 \\ \hline 4\,8\,4\,*\,*\,*\,* \end{array}$$

(iv)
$$\begin{array}{r} 1\,0\,0\,0\,0\,0\,0 \\ -\ \ \ \ \ *\,*\,*\,*\,1 \\ \hline \ \ \ *\,7\,0\,4\,2\,* \end{array}$$

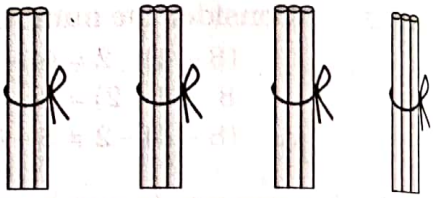
3. Find the difference:
(i) 463 – 9 (ii) 5632 – 99 (iii) 8640 – 999 (iv) 13006 – 9999

4. Find the difference between the smallest number of 7 digits and the largest number of 4 digits.
5. Ravi opened his account in a bank by depositing ₹ 136000. Next day he withdrew ₹ 73129 from it. How much money was left in his account?
6. Mrs Saxena withdrew ₹ 100000 from her bank account. She purchased a TV set for ₹ 38750, a refrigerator for ₹ 23890 and jewellery worth ₹ 35560. How much money was left with her?
7. The population of a town was 110500. In one year it increased by 3608 due to new births. However, 8973 persons died or left the town during the year. What was the population at the end of the year?
8. Find the whole number n when:
   (i) $n + 4 = 9$ (ii) $n + 35 = 101$ (iii) $n - 18 = 39$ (iv) $n - 20568 = 21403$

MULTIPLICATION OF WHOLE NUMBERS

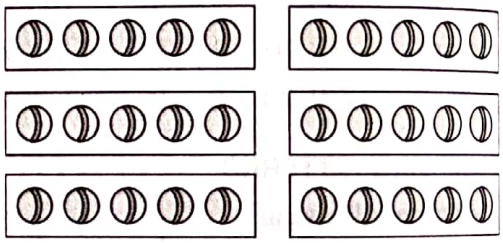
Let us consider 4 bundles, each consisting of 3 sticks.

Total number of sticks
$= 3 + 3 + 3 + 3 = 12.$
Also, we may write:
total number of sticks
= 4 times 3, written as 4×3.
$\therefore \quad 4 \times 3 = 12.$



Again, consider 6 packets of 5 balls each.

Total number of balls
$= 5 + 5 + 5 + 5 + 5 + 5 = 30.$
Also, we may write:
total number of balls
= 6 times 5, written as 6×5.
Therefore, $6 \times 5 = 30$.



It follows that *multiplication is repeated addition.*

If the numbers are small, we can perform the operation of multiplication mentally as above and find the product.

If the numbers are large, we multiply them using the multiplication tables about which you have learnt earlier.

However, we now list the various properties of multiplication on whole numbers. These properties will help us in finding easily the products of numbers, however large they may be.

PROPERTIES OF MULTIPLICATION OF WHOLE NUMBERS

(i) CLOSURE PROPERTY *If a and b are whole numbers, then $(a \times b)$ is also a whole number.*

EXAMPLES Let us take a few pairs of whole numbers and check in each case whether their product is a whole number.

One whole number	Another whole number	Product	Is the product a whole number?
9	8	$9 \times 8 = 72$	Yes
12	7	$12 \times 7 = 84$	Yes
16	10	$16 \times 10 = 160$	Yes

Thus, we see that if we multiply two whole numbers, the product is also a whole number.

(ii) COMMUTATIVE LAW *If a and b are any two whole numbers then* $(a \times b) = (b \times a)$.

EXAMPLES (i) $7 \times 5 = 35$ and $5 \times 7 = 35$.

Is $(7 \times 5) = (5 \times 7)$? Yes.

(ii) $19 \times 12 = 228$ and $12 \times 19 = 228$.

Is $(19 \times 12) = (12 \times 19)$? Yes.

In general, commutative law of multiplication holds in whole numbers.

(iii) MULTIPLICATIVE PROPERTY OF ZERO *For every whole number a, we have* $(a \times 0) = (0 \times a) = 0$.

EXAMPLES (i) $9 \times 0 = 0 \times 9 = 0$ (ii) $37 \times 0 = 0 \times 37 = 0$ (iii) $2386 \times 0 = 0 \times 2386 = 0$

(iv) MULTIPLICATIVE PROPERTY OF 1 *For any whole number a we have:* $(a \times 1) = (1 \times a) = a$.

EXAMPLES (i) $8 \times 1 = 1 \times 8 = 8$ (ii) $76 \times 1 = 1 \times 76 = 76$ (iii) $2345 \times 1 = 1 \times 2345 = 2345$

(v) ASSOCIATIVE LAW *If a, b, c are any whole numbers, then* $(a \times b) \times c = a \times (b \times c)$.

EXAMPLE Take the whole numbers 9, 7 and 10.

$(9 \times 7) \times 10 = 63 \times 10 = 630$.

$9 \times (7 \times 10) = 9 \times 70 = 630$.

$\therefore\ (9 \times 7) \times 10 = 9 \times (7 \times 10)$.

(vi) DISTRIBUTIVE LAW OF MULTIPLICATION OVER ADDITION *For any whole numbers a, b, c we have:* $a \times (b + c) = (a \times b) + (a \times c)$.

EXAMPLE Consider the whole numbers 16, 9 and 8.

$16 \times (9 + 8) = (16 \times 17) = 272$.

$(16 \times 9) + (16 \times 8) = (144 + 128) = 272$.

$\therefore\ 16 \times (9 + 8) = (16 \times 9) + (16 \times 8)$.

(vii) DISTRIBUTIVE LAW OF MULTIPLICATION OVER SUBTRACTION *For any whole numbers a, b, c we have:* $a \times (b - c) = (a \times b) - (a \times c)$.

EXAMPLE Consider the whole numbers 11, 6 and 4.

$11 \times (6 - 4) = (11 \times 2) = 22$.

$(11 \times 6) - (11 \times 4) = (66 - 44) = 22$.

$\therefore\ 11 \times (6 - 4) = (11 \times 6) - (11 \times 4)$.

SOLVED EXAMPLES

EXAMPLE 1. *Multiply* 197 *by* 54.

Solution We have:

$197 \times 54 = 197 \times (50 + 4)$

$= 197 \times 50 + 197 \times 4$ (by distributive law)

$= 9850 + 788 = 10638$.

Explanation In order to multiply 197 by 54, we find the products 197×4 and 197×50, and add them.

Thus, we have:

$$\begin{array}{r} 197 \\ \times\ 54 \\ \hline 788 \\ 9850 \\ \hline 10638 \end{array} \quad \begin{array}{l} \\ \\ \text{multiplication by 4} \\ \text{multiplication by 50} \\ \text{multiplication by 54} \end{array}$$

Therefore, we have: $197 \times 54 = 10638$.

EXAMPLE 2. *Multiply* 2056 *by* 87.

Solution

$$\begin{array}{r} 2056 \\ \times\ 87 \\ \hline 14392 \\ 164480 \\ \hline 178872 \end{array} \quad \begin{array}{l} \\ \\ \text{multiplication by 7} \\ \text{multiplication by 80} \\ \text{multiplication by 87} \end{array}$$

Therefore, we have: $2056 \times 87 = 178872$.

EXAMPLE 3. *Multiply* 1572 *by* 123.

Solution

$$\begin{array}{r} 1572 \\ \times\ 123 \\ \hline 4716 \\ 31440 \\ 157200 \\ \hline 193356 \end{array} \quad \begin{array}{l} \\ \\ \text{multiplication by 3} \\ \text{multiplication by 20} \\ \text{multiplication by 100} \\ \text{multiplication by 123} \end{array}$$

Therefore, we have: $1572 \times 123 = 193356$.

EXAMPLE 4. *Find the product:* 785×94.

Solution We have:

$$\begin{aligned} 785 \times 94 &= 785 \times (100 - 6) \\ &= 785 \times 100 - 785 \times 6 \\ &= 78500 - 4710 = 73790. \end{aligned}$$

EXAMPLE 5. *Find the value of* $968 \times 73 + 968 \times 27$.

Solution By the distributive law over addition, we have:

$$\begin{aligned} 968 \times 73 + 968 \times 27 &= 968 \times (73 + 27) \\ &= 968 \times 100 = 96800. \end{aligned}$$

EXAMPLE 6. *Find the value of* $1063 \times 127 - 1063 \times 27$.

Solution

$$\begin{aligned} 1063 \times 127 - 1063 \times 27 &= 1063 \times (127 - 27) \\ &= 1063 \times 100 = 106300. \end{aligned}$$

EXAMPLE 7. *Find the value of* $8937 \times 648 + 8937 \times 122 + 8937 \times 230$.

Solution

$$\begin{aligned} \text{The given expression} &= 8937 \times (648 + 122 + 230) \\ &= 8937 \times 1000 = 8937000. \end{aligned}$$

EXAMPLE 8. *Find the product:* $4 \times 2995 \times 250$.

Solution We have:

$$4 \times 2995 \times 250 = (4 \times 250) \times 2995$$

$= (1000 \times 2995) = 2995000.$

EXAMPLE 9. *Find the product* $37256 \times 25 \times 40$.

Solution We have:

$37256 \times 25 \times 40 = 37256 \times (25 \times 40)$

$= 37256 \times 1000 = 37256000.$

EXAMPLE 10. *Find each of the following products:*

(i) 30674×9 *(ii)* 4578×99 *(iii)* 23756×999

Solution We have:

(i) $30674 \times 9 = 30674 \times (10 - 1)$

$= (30674 \times 10) - (30674 \times 1)$

$= (306740 - 30674) = 276066.$

(ii) $4578 \times 99 = 4578 \times (100 - 1)$

$= (4578 \times 100) - (4578 \times 1)$

$= (457800 - 4578) = 453222.$

(iii) $23756 \times 999 = 23756 \times (1000 - 1)$

$= (23756 \times 1000) - (23756 \times 1)$

$= (23756000 - 23756) = 23732244.$

EXERCISE 3D

1. Fill in the blanks to make each of the following a true statement:

(i) $246 \times 1 = \ldots\ldots$ (ii) $1369 \times 0 = \ldots\ldots$

(iii) $593 \times 188 = 188 \times \ldots\ldots$ (iv) $286 \times 753 = \ldots\ldots \times 286$

(v) $38 \times (91 \times 37) = \ldots\ldots \times (38 \times 37)$

(vi) $13 \times 100 \times \ldots\ldots = 1300000$

(vii) $59 \times 66 + 59 \times 34 = 59 \times (\ldots\ldots + \ldots\ldots)$

(viii) $68 \times 95 = 68 \times 100 - 68 \times \ldots\ldots$

2. State the property used in each of the following statements:

(i) $19 \times 17 = 17 \times 19$ (ii) (16×32) is a whole number

(iii) $(29 \times 36) \times 18 = 29 \times (36 \times 18)$ (iv) $1480 \times 1 = 1480$

(v) $1732 \times 0 = 0$ (vi) $72 \times 98 + 72 \times 2 = 72 \times (98 + 2)$

(vii) $63 \times 126 - 63 \times 26 = 63 \times (126 - 26)$

3. Find the value of each of the following using various properties:

(i) $647 \times 13 + 647 \times 7$ (ii) $8759 \times 94 + 8759 \times 6$

(iii) $7459 \times 999 + 7459$ (iv) $9870 \times 561 - 9870 \times 461$

(v) $569 \times 17 + 569 \times 13 + 569 \times 70$ (vi) $16825 \times 16825 - 16825 \times 6825$

4. Determine each of the following products by suitable rearrangements:

(i) $2 \times 1658 \times 50$ (ii) $4 \times 927 \times 25$ (iii) $625 \times 20 \times 8 \times 50$

(iv) $574 \times 625 \times 16$ (v) $250 \times 60 \times 50 \times 8$ (vi) $8 \times 125 \times 40 \times 25$

5. Find each of the following products, using distributive laws:

(i) 740×105 (ii) 245×1008 (iii) 947×96

(iv) 996×367 (v) 472×1097 (vi) 580×64

(vii) 439×997 (viii) 1553×198

6. Find each of the following products, using distributive laws:

(i) 3576×9 (ii) 847×99 (iii) 2437×999

7. Find the products:

(i) $\begin{array}{r} 458 \\ \times\ 67 \\ \hline \end{array}$ (ii) $\begin{array}{r} 3709 \\ \times\ 89 \\ \hline \end{array}$ (iii) $\begin{array}{r} 4617 \\ \times\ 234 \\ \hline \end{array}$ (iv) $\begin{array}{r} 15208 \\ \times\ 542 \\ \hline \end{array}$

8. Find the product of the largest 3-digit number and the largest 5-digit number.
Hint. $999 \times 99999 = 999 \times (100000 - 1)$. *Now, use distributive law.*

9. A car moves at a uniform speed of 75 km per hour. How much distance will it cover in 98 hours?

10. A dealer purchased 139 VCRs. If the cost of each set is ₹ 24350, find the cost of all the sets together.

11. A housing society constructed 197 houses. If the cost of construction for each house is ₹ 450000, what is the total cost for all the houses?

12. 50 chairs and 30 blackboards were purchased for a school. If each chair costs ₹ 1065 and each blackboard costs ₹ 1645, find the total amount of the bill.

13. There are six sections of Class VI in a school and there are 45 students in each section. If the monthly charges from each student be ₹ 1650, find the total monthly collection from Class VI.

14. The product of two whole numbers is zero. What do you conclude?

15. Fill in the blanks:
(i) Sum of two odd numbers is an ..... number.
(ii) Product of two odd numbers is an ..... number.
(iii) $a \neq 0$ and $a \times a = a \Rightarrow a = ?$

DIVISION IN WHOLE NUMBERS

Division is the inverse operation of multiplication.

Let a and b be two whole numbers. Dividing a by b means finding a whole number c such that $b \times c = a$ *and we write,* $\boldsymbol{a \div b = c}$.

Thus, $a \div b = c \Rightarrow \frac{a}{b} = c \Rightarrow a = b \times c$.

EXAMPLES Dividing 48 by 8 is the same as finding a whole number which when multiplied by 8 gives 48.
Clearly, such a number is 6, as $8 \times 6 = 48$.
Similarly, we have:
$63 \div 9 = 7$, $84 \div 14 = 6$, etc.

DIVISION ALGORITHM Suppose 75 is divided by 9, then the quotient is 8 and the remainder is 3.

$$\begin{array}{r} 8 \\ 9\overline{)75} \\ -72 \\ \hline 3 \end{array}$$

Clearly, $75 = (9 \times 8) + 3$.

In general, *let a and b be two given whole numbers such that* $a > b$. *On dividing a by b, let q be the quotient and r be the remainder.*

Then, we have: $a = bq + r$, where $0 \leq r < b$.

This result is known as **division algorithm.**

Thus, ***dividend = (divisor × quotient) + remainder.***

EVEN AND ODD WHOLE NUMBERS *A whole number divisible by 2 is called an even number;*

e.g., 0, 2, 4, 6, 8, etc., are all even numbers.

A whole number which is not divisible by 2 is called an odd number;

e.g., 1, 3, 5, 7, 9, etc., are all odd numbers.

SOLVED EXAMPLES

EXAMPLE 1. *Find the number which when divided by 53 gives 8 as quotient and 5 as remainder.*

Solution Given: divisor = 53, quotient = 8 and remainder = 5.

By division algorithm, we have:

$$\text{dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}$$
$$= (53 \times 8) + 5$$
$$= (424 + 5) = 429.$$

Hence, the required number is 429.

EXAMPLE 2. *Divide 535 by 31 and check the result by the division algorithm.*

Solution By actual division, we have:

$$\begin{array}{r} 31 \overline{)\,535\,(\,17} \\ -\,31 \\ \hline 225 \\ -\,217 \\ \hline 8 \end{array}$$

$\therefore$ dividend = 535, divisor = 31, quotient = 17 and remainder = 8.

CHECK $(31 \times 17) + 8 = 527 + 8 = 535$.

Hence, the above result is correct.

EXAMPLE 3. *Divide 53068 by 257 and check the result by the division algorithm.*

Solution By actual division, we have:

$$\begin{array}{r} 257 \overline{)\,53068\,(\,206} \\ -\,514 \\ \hline 1668 \\ -\,1542 \\ \hline 126 \end{array}$$

$\therefore$ dividend = 53068, divisor = 257, quotient = 206 and remainder = 126.

CHECK $(257 \times 206) + 126 = 52942 + 126 = 53068$.

Hence, the above result is correct.

PROPERTIES OF DIVISION

(i) *If a and b are nonzero whole numbers, then $a \div b$ is not always a whole number.*

EXAMPLE We know that 7 and 2 are whole numbers.

But, $7 \div 2$ is not a whole number.

(ii) DIVISION BY 0 *If a is a whole number, then $a \div 0$ is meaningless.*

(iii) *If a is a nonzero whole number, then* $0 \div a = 0$.

EXAMPLES (i) $0 \div 3 = 0$ (ii) $0 \div 57 = 0$, etc.

EXERCISE 3E

1. Divide and check your answer by the corresponding multiplication in each of the following:
   (i) $1936 \div 16$ (ii) $19881 \div 47$ (iii) $257796 \div 341$
   (iv) $612846 \div 582$ (v) $34419 \div 149$ (vi) $39039 \div 1001$
2. Divide, and find out the quotient and remainder. Check your answer.
   (i) $6971 \div 47$ (ii) $4178 \div 35$ (iii) $36195 \div 153$
   (iv) $93575 \div 400$ (v) $23025 \div 1000$ (vi) $16135 \div 875$
3. Find the value of
   (i) $65007 \div 1$ (ii) $0 \div 879$
   (iii) $981 + 5720 \div 10$ (iv) $1507 - (625 \div 25)$
   (v) $32277 \div (648 - 39)$ (vi) $(1573 \div 1573) - (1573 \div 1573)$
4. Find a whole number n such that $n \div n = n$.
5. The product of two numbers is 504347. If one of the numbers is 317, find the other.
6. On dividing 59761 by a certain number, the quotient is 189 and the remainder is 37. Find the divisor.
7. On dividing 55390 by 299, the remainder is 75. Find the quotient using the division algorithm.
8. What least number must be subtracted from 13601 to get a number exactly divisible by 87?
9. What least number must be added to 1056 to get a number exactly divisible by 23?
10. Find the largest 4-digit number divisible by 16.
11. Divide the largest 5 digit number by 653. Check your answer by the division algorithm.
12. Find the least 6-digit number exactly divisible by 83.
13. 1 dozen bananas cost ₹ 29. How many dozens can be purchased for ₹ 1392?
14. 19625 trees have been equally planted in 157 rows. Find the number of trees in each row.
15. The population of a town is 517530. If one out of every 15 is reported to be literate, find how many literate persons are there in the town.
16. The cost price of 23 colour television sets is ₹ 570055. Determine the cost price of each TV set if each costs the same.

EXERCISE 3F

OBJECTIVE QUESTIONS

Mark (✓) against the correct answer in each of the following:

1. The smallest whole number is
   (a) 1 (b) 0 (c) 2 (d) none of these
2. The least number of 4 digits which is exactly divisible by 9 is
   (a) 1018 (b) 1026 (c) 1009 (d) 1008

3. The largest number of 6 digits which is exactly divisible by 16 is
(a) 999980 (b) 999982 (c) 999984 (d) 999964

4. What least number should be subtracted from 10004 to get a number exactly divisible by 12?
(a) 4 (b) 6 (c) 8 (d) 20

5. What least number should be added to 10056 to get a number exactly divisible by 23?
(a) 5 (b) 18 (c) 13 (d) 10

6. What whole number is nearest to 457 which is divisible by 11?
(a) 450 (b) 451 (c) 460 (d) 462

7. How many whole numbers are there between 1018 and 1203?
(a) 185 (b) 186 (c) 184 (d) none of these

8. A number when divided by 46 gives 11 as quotient and 15 as remainder. The number is
(a) 491 (b) 521 (c) 701 (d) 679

9. In a division sum, we have dividend = 199, quotient = 16 and remainder = 7. The divisor is
(a) 11 (b) 23 (c) 12 (d) none of these

10. $7589 - ? = 3434$
(a) 11023 (b) 4245 (c) 4155 (d) none of these

11. $587 \times 99 = ?$
(a) 57213 (b) 58513 (c) 58113 (d) 56413

12. $4 \times 538 \times 25 = ?$
(a) 32280 (b) 26900 (c) 53800 (d) 10760

13. $24679 \times 92 + 24679 \times 8 = ?$
(a) 493580 (b) 1233950 (c) 2467900 (d) none of these

14. $1625 \times 1625 - 1625 \times 625 = ?$
(a) 1625000 (b) 162500 (c) 325000 (d) 812500

15. $1568 \times 185 - 1568 \times 85 = ?$
(a) 7840 (b) 15680 (c) 156800 (d) none of these

16. $(888 + 777 + 555) = (111 \times ?)$
(a) 120 (b) 280 (c) 20 (d) 140

17. The sum of two odd numbers is
(a) an odd number (b) an even number (c) a prime number (d) a multiple of 3

18. The product of two odd numbers is
(a) an odd number (b) an even number (c) a prime number (d) none of these

19. If a is a whole number such that $a + a = a$, then $a = ?$
(a) 1 (b) 2 (c) 3 (d) none of these

20. The predecessor of 10000 is
(a) 10001 (b) 9999 (c) none of these

21. The successor of 1001 is
(a) 1000 (b) 1002 (c) none of these

22. The smallest even whole number is
(a) 0 (b) 2 (c) none of these

Things to Remember

1. *If a, b, c are whole numbers then*
   (i) $a + b$ *is a whole number*
   (ii) $a \times b$ *is a whole number*
   (iii) $a - b$ *is not necessarily a whole number*
   (iv) $a \div b$ *is not necessarily a whole number*
   (v) $a + b = b + a$
   (vi) $a \times b = b \times a$
   (vii) $(a + b) + c = a + (b + c)$
   (viii) $(a \times b) \times c = a \times (b \times c)$
   (ix) $a \times (b + c) = a \times b + a \times c$
   (x) $a \times (b - c) = a \times b - a \times c$
   (xi) $a + 0 = 0 + a = a$ *and* $a \times 0 = 0 \times a = 0$
   (xii) $a \times 1 = 1 \times a = a$
2. *In general,* $(a - b) - c \neq a - (b - c)$.
3. *If a is the dividend, b (where* $b \neq 0$*) is the divisor, q is the quotient and r is the remainder then* $a = bq + r$.
4. *If a is a nonzero whole number, then* $a \div 0$ *is not defined and* $0 \div a = 0$.

TEST PAPER-3

A. **1.** How many whole numbers are there between 1064 and 1201?

2. Fill in the blanks.

$$\begin{array}{r} 1000000 \\ -\ ****1 \\ \hline *7042* \end{array}$$

3. Use distributive law to find the value of

$1063 \times 128 - 1063 \times 28$.

4. Find the product of the largest 5-digit number and the largest 3-digit number using distributive law.

5. Divide 53968 by 267 and check the result by the division algorithm.

6. Find the largest 6-digit number divisible by 16.

7. The cost price of 23 TV sets is ₹ 570055. Find the cost of each such set.

8. What least number must be subtracted from 13801 to get a number exactly divisible by 87?

B. ***Mark (✓) against the correct answer in each of the following:***

9. The value of $(89 \times 76 + 89 \times 24)$ is

(a) 890 (b) 8900 (c) 89000 (d) 10420

10. On dividing a number by 53 we get 8 as quotient and 5 as remainder. The number is

(a) 419 (b) 423 (c) 429 (d) none of these

11. The whole number which has no predecessor is

(a) 1 (b) 0 (c) 2 (d) none of these

12. $67 + 33 = 33 + 67$ is an example of

(a) closure property (b) associative property

(c) commutative property (d) distributive property

13. Additive inverse of 36 is

(a) $\frac{1}{36}$ (b) 0 (c) –36 (d) none of these

14. Which of the following is not zero?

(a) 0×0 (b) $\frac{0}{2}$ (c) $\frac{(8-8)}{2}$ (d) $2 + 0$

15. The predecessor of the smallest 3-digit number is

(a) 999 (b) 100 (c) 101 (d) 99

16. The number of whole numbers between the smallest whole number and the greatest 2-digit number is

(a) 88 (b) 98 (c) 99 (d) 101

C. **17.** ***Fill in the blanks.***

(i) The smallest natural number is ..... .

(ii) The smallest whole number is ..... .

(iii) Division by ..... is not defined.

(iv) ..... is a whole number which is not a natural number.

(v) ..... is the multiplicative identity in whole numbers.

D. 18. ***Write 'T' for true and 'F' for false in each of the following:***

(i) 0 is the smallest natural number.

(ii) Every natural number is a whole number.

(iii) Every whole number is a natural number.

(iv) 1 has no predecessor in whole numbers.

E. 19. ***Match the following columns on whole numbers:***

Column A	Column B
(a) $137 + 63 = 63 + 137$	(i) Associativity of multiplication
(b) (16×25) is a whole number	(ii) Commutativity of multiplication
(c) $365 \times 18 = 18 \times 365$	(iii) Distributive law of multiplication over addition
(d) $(86 \times 14) \times 25 = 86 \times (14 \times 25)$	(iv) Commutativity of addition
(e) $23 \times (80 + 5) = (23 \times 80) + (23 \times 5)$	(v) Closure property for multiplication