Mathematics

For Class 8

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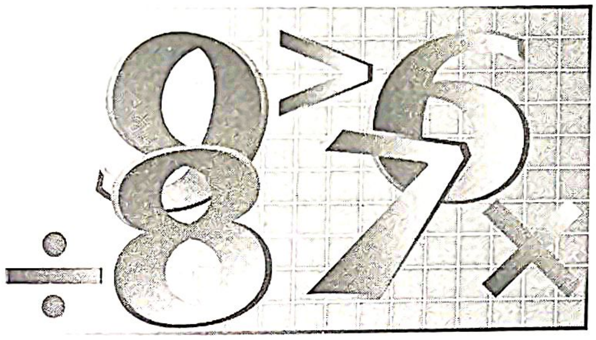
Rational Numbers



In our previous class we have studied about natural numbers, whole numbers, integers and fractions. We have also studied about various operations on rational numbers. In this chapter we shall study the properties of these operations on rational numbers.

Rational numbers *The numbers of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called rational numbers.*

EXAMPLES *Each of the numbers $\frac{5}{8}, \frac{-3}{14}, \frac{7}{-15}$ and $\frac{-6}{-11}$ is a rational number.*

Positive rationals *A rational number is said to be positive if its numerator and denominator are either both positive or both negative.*

Thus, $\frac{5}{7}$ and $\frac{-2}{-3}$ are both positive rationals.

Negative rationals *A rational number is said to be negative if its numerator and denominator are of opposite signs.*

Thus, $\frac{-4}{9}$ and $\frac{5}{-12}$ are both negative rationals.

Three Properties of Rational Numbers:

Property 1. *If $\frac{a}{b}$ is a rational number and m is a nonzero integer then $\frac{a}{b} = \frac{a \times m}{b \times m}$.*

EXAMPLE $\frac{-3}{4} = \frac{(-3) \times 2}{4 \times 2} = \frac{(-3) \times 3}{4 \times 3} = \frac{(-3) \times 4}{4 \times 4} = \ldots$

$\Rightarrow \frac{-3}{4} = \frac{-6}{8} = \frac{-9}{12} = \frac{-12}{16} = \ldots$

Such rational numbers are called ***equivalent rational numbers.***

Property 2. *If $\frac{a}{b}$ is a rational number and m is a common divisor of a and b, then $\frac{a}{b} = \frac{a \div m}{b \div m}$.*

Thus, we can write, $\frac{-32}{40} = \frac{-32 \div 8}{40 \div 8} = \frac{-4}{5}$.

Standard form of a rational number

A rational number $\frac{a}{b}$ is said to be in standard form if a and b are integers having no common divisor other than 1 and b is positive.

EXAMPLE 1. *Express $\frac{33}{-44}$ in standard form.*

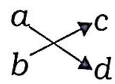
Solution $\frac{33}{-44} = \frac{33 \times (-1)}{(-44) \times (-1)} = \frac{-33}{44}$.

The greatest common divisor of 33 and 44 is 11.

$\therefore \quad \frac{-33}{44} = \frac{(-33) \div 11}{44 \div 11} = \frac{-3}{4}$.

Hence, $\frac{-33}{44} = \frac{-3}{4}$ (in standard form).

Property 3. *Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers. Then,* $\frac{a}{b} = \frac{c}{d} \Leftrightarrow (a \times d) = (b \times c)$.



$a \times d = b \times c$

COMPARISON OF RATIONAL NUMBERS

It is clear that:

(i) every positive rational number is greater than 0,
(ii) every negative rational number is less than 0.

GENERAL METHOD OF COMPARING RATIONAL NUMBERS

Step 1: Express each of the two given rational numbers with positive denominator.

Step 2: Take the LCM of these positive denominators.

Step 3: Express each rational number (obtained in Step 1) with this LCM as the common denominator.

Step 4: The number having the greater numerator is greater.

EXAMPLE 2. *Which of the numbers $\frac{3}{-4}$ or $\frac{-5}{6}$ is greater?*

Solution First we write each of the given numbers with positive denominator.

One number $= \frac{3}{-4} = \frac{3 \times (-1)}{(-4) \times (-1)} = \frac{-3}{4}$.

The other number $= \frac{-5}{6}$.

LCM of 4 and 6 = 12.

$\therefore \quad \frac{-3}{4} = \frac{(-3) \times 3}{4 \times 3} = \frac{-9}{12}$ and $\frac{-5}{6} = \frac{(-5) \times 2}{6 \times 2} = \frac{-10}{12}$.

Clearly, $-9 > -10$. $\therefore \quad \frac{-9}{12} > \frac{-10}{12}$.

Hence, $\frac{-3}{4} > \frac{-5}{6}$, i.e., $\frac{3}{-4} > \frac{-5}{6}$.

EXAMPLE 3. *Arrange the numbers $\frac{-3}{5}, \frac{7}{-10}$ and $\frac{-5}{8}$ in ascending order.*

Solution First we write each of the given numbers with positive denominator. We have:

$\frac{7}{-10} = \frac{7 \times (-1)}{(-10) \times (-1)} = \frac{-7}{10}$.

Thus, the given numbers are $\frac{-3}{5}, \frac{-7}{10}$ and $\frac{-5}{8}$.

LCM of 5, 10 and 8 is 40.

Now, $\frac{-3}{5} = \frac{(-3)\times 8}{5\times 8} = \frac{-24}{40}$; $\frac{-7}{10} = \frac{(-7)\times 4}{10\times 4} = \frac{-28}{40}$ and $\frac{-5}{8} = \frac{(-5)\times 5}{8\times 5} = \frac{-25}{40}$.

Clearly, $\frac{-28}{40} < \frac{-25}{40} < \frac{-24}{40}$.

Hence, $\frac{-7}{10} < \frac{-5}{8} < \frac{-3}{5}$, i.e., $\frac{7}{-10} < \frac{-5}{8} < \frac{-3}{5}$.

EXERCISE 1A

1. Express $\frac{-3}{5}$ as a rational number with denominator

   (i) 20 (ii) −30 (iii) 35 (iv) −40

2. Express $\frac{-42}{98}$ as a rational number with denominator 7.

3. Express $\frac{-48}{60}$ as a rational number with denominator 5.

4. Express each of the following rational numbers in standard form:

   (i) $\frac{-12}{30}$ (ii) $\frac{-14}{49}$ (iii) $\frac{24}{-64}$ (iv) $\frac{-36}{-63}$

5. Which of the two rational numbers is greater in the given pair?

   (i) $\frac{3}{8}$ or 0 (ii) $\frac{-2}{9}$ or 0 (iii) $\frac{-3}{4}$ or $\frac{1}{4}$

   (iv) $\frac{-5}{7}$ or $\frac{-4}{7}$ (v) $\frac{2}{3}$ or $\frac{3}{4}$ (vi) $\frac{-1}{2}$ or −1

6. Which of the two rational numbers is greater in the given pair?

   (i) $\frac{-4}{3}$ or $\frac{-8}{7}$ (ii) $\frac{7}{-9}$ or $\frac{-5}{8}$ (iii) $\frac{-1}{3}$ or $\frac{4}{-5}$

   (iv) $\frac{9}{-13}$ or $\frac{7}{-12}$ (v) $\frac{4}{-5}$ or $\frac{-7}{10}$ (vi) $\frac{-12}{5}$ or −3

7. Fill in the blanks with the correct symbol out of >, = and <:

   (i) $\frac{-3}{7} \ldots\ldots \frac{6}{-13}$ (ii) $\frac{5}{-13} \ldots\ldots \frac{-35}{91}$ (iii) $-2 \ldots\ldots \frac{-13}{5}$

   (iv) $\frac{-2}{3} \ldots\ldots \frac{5}{-8}$ (v) $0 \ldots\ldots \frac{-3}{-5}$ (vi) $\frac{-8}{9} \ldots\ldots \frac{-9}{10}$

8. Arrange the following rational numbers in ascending order:

   (i) $\frac{4}{-9}, \frac{-5}{12}, \frac{7}{-18}, \frac{-2}{3}$ (ii) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$

   (iii) $\frac{3}{-5}, \frac{-7}{10}, \frac{-11}{15}, \frac{-13}{20}$ (iv) $\frac{-4}{7}, \frac{-9}{14}, \frac{13}{-28}, \frac{-23}{42}$

9. Arrange the following rational numbers in descending order:

   (i) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$ (ii) $\frac{-3}{10}, \frac{7}{-15}, \frac{-11}{20}, \frac{17}{-30}$

   (iii) $\frac{-5}{6}, \frac{-7}{12}, \frac{-13}{18}, \frac{23}{-24}$ (iv) $\frac{-10}{11}, \frac{-19}{22}, \frac{-23}{33}, \frac{-39}{44}$

10. Which of the following statements are true and which are false?

   (i) Every whole number is a rational number.

   (ii) Every integer is a rational number.

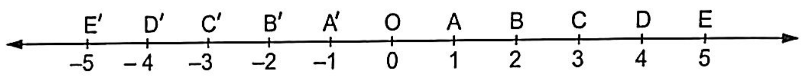
(iii) 0 is a whole number but it is not a rational number.

REPRESENTATION OF RATIONAL NUMBERS ON THE REAL LINE

In the previous class we have learnt how to represent integers on the number line.

Let us review it.

Draw any line. Take a point *O* on it. Call it 0 (zero). Set off equal distances on the right as well as on the left of *O*. Such a distance is known as a unit length. Clearly, the points *A*, *B*, *C*, *D* and *E* represent the integers 1, 2, 3, 4 and 5 respectively and the points *A′*, *B′*, *C′*, *D′* and *E′* represent the integers –1, –2, –3, –4 and –5 respectively.



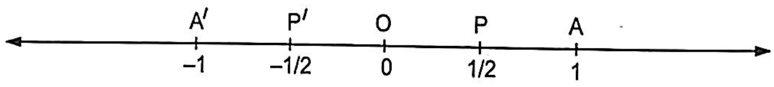
Thus, we may represent any integer by a point on the number line. Clearly, every positive integer lies to the right of *O* and every negative integer lies to the left of *O*.

Similarly we can represent rational numbers.

Consider the following examples.

EXAMPLE 1. *Represent* $\frac{1}{2}$ *and* $-\frac{1}{2}$ *on the number line.*

Solution Draw a line. Take a point *O* on it . Let it represent 0. Set off unit lengths *OA* and *OA′* to the right and to the left of *O* respectively.
Then, *A* represents the integer 1 and *A′* represents the integer –1.

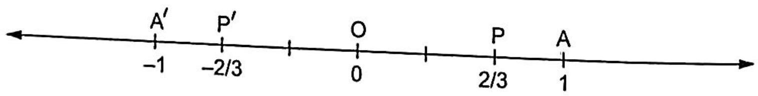


Now, divide *OA* into two equal parts. Let *OP* be the first part out of these two parts. Then, the point *P* represents the rational number $\frac{1}{2}$.

Again, divide *OA′* into two equal parts. Let *OP′* be the first part out of these 2 parts. Then, the point *P′* represents the rational number $-\frac{1}{2}$.

EXAMPLE 2. *Represent* $\frac{2}{3}$ *and* $-\frac{2}{3}$ *on the number line.*

Solution Draw a line. Take a point *O* on it. Let it represent 0. From *O* set off unit distances *OA* and *OA′* to the right and left of *O* respectively.
Divide *OA* into 3 equal parts. Let *OP* be the segment showing 2 parts out of 3. Then, the point *P* represents the rational number $\frac{2}{3}$.



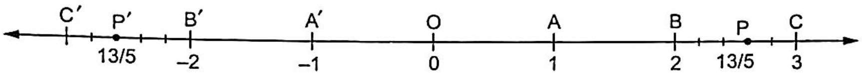
Again divide *OA′* into 3 equal parts. Let *OP′* be the segment consisting of 2 parts out of these 3 parts. Then, the point *P′* represents the rational number $-\frac{2}{3}$.

EXAMPLE 3. *Represent $\frac{13}{5}$ and $\frac{-13}{5}$ on the number line.*

Solution Draw a line. Take a point O on it. Let it represent 0.

Now, $\frac{13}{5} = 2\frac{3}{5} = 2 + \frac{3}{5}$.

From O, set off unit distances OA, AB and BC to the right of O. Clearly, the points A, B and C represent the integers 1, 2 and 3 respectively. Now, take 2 units OA and AB, and divide the third unit BC into 5 equal parts. Take 3 parts out of these 5 parts to reach at a point P. Then, the point P represents the rational number $\frac{13}{5}$.



Again, from O, set off unit distances to the left. Let these segments be OA', $A'B'$, $B'C'$, etc. Then, clearly the points A', B' and C' represent the integers $-1, -2, -3$ respectively.

Now, $\frac{-13}{5} = -\left(2 + \frac{3}{5}\right)$.

Take 2 full unit lengths to the left of O. Divide the third unit $B'C'$ into 5 equal parts. Take 3 parts out of these 5 parts to reach a point P'.

Then, the point P' represents the rational number $\frac{-13}{5}$.

Thus, we can represent every rational number by a point on the number line.

EXERCISE 1B

1. Represent each of the following numbers on the number line:

(i) $\frac{1}{3}$ (ii) $\frac{2}{7}$ (iii) $1\frac{3}{4}$ (iv) $2\frac{2}{5}$

(v) $3\frac{1}{2}$ (vi) $5\frac{5}{7}$ (vii) $4\frac{2}{3}$ (viii) 8

2. Represent each of the following numbers on the number line:

(i) $\frac{-1}{3}$ (ii) $\frac{-3}{4}$ (iii) $-1\frac{2}{3}$ (iv) $-3\frac{1}{7}$

(v) $-4\frac{3}{5}$ (vi) $-2\frac{5}{6}$ (vii) -3 (viii) $-2\frac{7}{8}$

3. Which of the following statements are true and which are false?

(i) $\frac{-3}{5}$ lies to the left of 0 on the number line.

(ii) $\frac{-12}{7}$ lies to the right of 0 on the number line.

(iii) The rational numbers $\frac{1}{3}$ and $\frac{-5}{2}$ are on opposite sides of 0 on the number line.

(iv) The rational number $\frac{-18}{-13}$ lies to the left of 0 on the number line.

ADDITION OF RATIONAL NUMBERS

If two rational numbers are to be added, we should convert each of them into a rational number with positive denominator.

CASE 1. **When Given Numbers have Same Denominator:**

In this case, we define $\left(\frac{a}{b}+\frac{c}{b}\right)=\frac{(a+c)}{b}$.

EXAMPLE 1. *Find the sum:*

(i) $\frac{7}{9}+\frac{-11}{9}$ (ii) $\frac{8}{-11}+\frac{3}{11}$

Solution We have:

(i) $\frac{7}{9}+\frac{-11}{9}=\frac{7+(-11)}{9}=\frac{-4}{9}$.

(ii) $\frac{8}{-11}=\frac{8\times(-1)}{(-11)\times(-1)}=\frac{-8}{11}$.

$\therefore \left(\frac{8}{-11}+\frac{3}{11}\right)=\left(\frac{-8}{11}+\frac{3}{11}\right)=\frac{(-8)+3}{11}=\frac{-5}{11}$.

CASE 2. **When Denominators of Given Numbers are Unequal:**

Method *In this case we take the LCM of their denominators and express each of the given numbers with this LCM as the common denominator. Now, we add these numbers as shown above.*

EXAMPLE 2. *Find the sum:* $\frac{-5}{6}+\frac{4}{9}$.

Solution The denominators of the given rational numbers are 6 and 9 respectively.

LCM of 6 and 9 = $(3\times2\times3)=18$.

3	6, 9
	2, 3

Now, $\frac{-5}{6}=\frac{(-5)\times3}{6\times3}=\frac{-15}{18}$ and $\frac{4}{9}=\frac{4\times2}{9\times2}=\frac{8}{18}$.

$\therefore \left(\frac{-5}{6}+\frac{4}{9}\right)=\left(\frac{-15}{18}+\frac{8}{18}\right)=\frac{(-15)+8}{18}=\frac{-7}{18}$.

Short-Cut Method

EXAMPLE 3. *Find the sum:* $\frac{-9}{16}+\frac{5}{12}$.

Solution LCM of 16 and 12 = $(4\times4\times3)=48$.

4	16, 12
	4, 3

$\therefore \frac{-9}{16}+\frac{5}{12}=\frac{3\times(-9)+4\times5}{48}=\frac{(-27)+20}{48}=\frac{-7}{48}$.

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Property 1 (Closure Property): *The sum of two rational numbers is always a rational number.*

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

EXAMPLES (i) Consider the rational numbers $\frac{1}{3}$ and $\frac{3}{4}$. Then,

$\left(\frac{1}{3}+\frac{3}{4}\right)=\frac{(4+9)}{12}=\frac{13}{12}$, which is a rational number.

(ii) Consider the rational numbers $\frac{-2}{3}$ and $\frac{4}{5}$. Then,

$$\left(\frac{-2}{3}+\frac{4}{5}\right)=\frac{(-10+12)}{15}=\frac{2}{15},$$ which is a rational number.

(iii) Consider the rational numbers $\frac{-5}{12}$ and $\frac{-1}{4}$. Then,

$$\left(\frac{-5}{12}+\frac{-1}{4}\right)=\frac{\{-5+(-3)\}}{12}=\frac{-8}{12}=\frac{-2}{3},$$ which is a rational number.

Property 2 (Commutative Law): *Two rational numbers can be added in any order.*

Thus for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$$\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right).$$

EXAMPLES (i) $\left(\frac{1}{2}+\frac{3}{4}\right)=\frac{(2+3)}{4}=\frac{5}{4}$ and $\left(\frac{3}{4}+\frac{1}{2}\right)=\frac{(3+2)}{4}=\frac{5}{4}$.

$\therefore \quad \left(\frac{1}{2}+\frac{3}{4}\right)=\left(\frac{3}{4}+\frac{1}{2}\right).$

(ii) $\left\{\frac{3}{8}+\frac{-5}{6}\right\}=\frac{\{9+(-20)\}}{24}=\frac{-11}{24}$ and $\left\{\frac{-5}{6}+\frac{3}{8}\right\}=\frac{\{-20+9\}}{24}=\frac{-11}{24}$.

$\therefore \quad \left(\frac{3}{8}+\frac{-5}{6}\right)=\left(\frac{-5}{6}+\frac{3}{8}\right).$

(iii) $\left(\frac{-1}{2}+\frac{-2}{3}\right)=\frac{\{(-3)+(-4)\}}{6}=\frac{-7}{6}$ and $\left(\frac{-2}{3}+\frac{-1}{2}\right)=\frac{\{(-4)+(-3)\}}{6}=\frac{-7}{6}$.

$\therefore \quad \left(\frac{-1}{2}+\frac{-2}{3}\right)=\left(\frac{-2}{3}+\frac{-1}{2}\right).$

Property 3 (Associative Law): *While adding three rational numbers, they can be grouped in any order.*

Thus, for any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right).$$

EXAMPLE Consider three rationals $\frac{-2}{3}$, $\frac{5}{7}$ and $\frac{1}{6}$. Then,

$$\left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{(-14+15)}{21}+\frac{1}{6}\right\}=\left(\frac{1}{21}+\frac{1}{6}\right)=\frac{(2+7)}{42}=\frac{9}{42}=\frac{3}{14}$$

and $$\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}=\left\{\frac{-2}{3}+\frac{(30+7)}{42}\right\}=\left(\frac{-2}{3}+\frac{37}{42}\right)=\frac{(-28+37)}{42}=\frac{9}{42}=\frac{3}{14}.$$

$$\therefore \quad \left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}.$$

Property 4 (Existence of Additive Identity): *0 is a rational number such that the sum of any rational number and 0 is the rational number itself.*

Thus, $\left(\frac{a}{b}+0\right)=\left(0+\frac{a}{b}\right)=\frac{a}{b}$, for every rational number $\frac{a}{b}$.

0 is called the ***additive identity*** for rationals.

EXAMPLES (i) $\left(\frac{3}{5}+0\right)=\left(\frac{3}{5}+\frac{0}{5}\right)=\frac{(3+0)}{5}=\frac{3}{5}$ and similarly, $\left(0+\frac{3}{5}\right)=\frac{3}{5}$.

$\therefore \quad \left(\frac{3}{5}+0\right)=\left(0+\frac{3}{5}\right)=\frac{3}{5}$.

(ii) $\left(\frac{-2}{3}+0\right)=\left(\frac{-2}{3}+\frac{0}{3}\right)=\frac{(-2+0)}{3}=\frac{-2}{3}$ and similarly, $\left(0+\frac{-2}{3}\right)=\frac{-2}{3}$.

$\therefore \quad \left(\frac{-2}{3}+0\right)=\left(0+\frac{-2}{3}\right)=\frac{-2}{3}$.

Property 5 (Existence of Additive Inverse): *For every rational number* $\frac{a}{b}$, *there exists a rational number* $\frac{-a}{b}$ *such that* $\left(\frac{a}{b}+\frac{-a}{b}\right)=\frac{\{a+(-a)\}}{b}=\frac{0}{b}=0$ *and similarly,* $\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.

Thus, $\left(\frac{a}{b}+\frac{-a}{b}\right)=\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.

$\frac{-a}{b}$ is called the ***additive inverse*** of $\frac{a}{b}$.

EXAMPLE $\left(\frac{4}{7}+\frac{-4}{7}\right)=\frac{\{4+(-4)\}}{7}=\frac{0}{7}=0$ and similarly, $\left(\frac{-4}{7}+\frac{4}{7}\right)=0$.

$\therefore \quad \left(\frac{4}{7}+\frac{-4}{7}\right)=\left(\frac{-4}{7}+\frac{4}{7}\right)=0$.

Thus, $\frac{4}{7}$ and $\frac{-4}{7}$ are additive inverses of each other.

SUBTRACTION OF RATIONAL NUMBERS

For rational numbers $\frac{a}{b}$ *and* $\frac{c}{d}$, *we define:*

$$\left(\frac{a}{b}-\frac{c}{d}\right)=\frac{a}{b}+\left(\frac{-c}{d}\right)=\frac{a}{b}+\left(\textbf{additive inverse of } \frac{c}{d}\right).$$

SOLVED EXAMPLES

EXAMPLE 1. *Find the additive inverse of:*

(i) $\frac{5}{9}$ *(ii)* $\frac{-15}{8}$ *(iii)* $\frac{9}{-11}$ *(iv)* $\frac{-6}{-7}$

Solution (i) Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

(ii) Additive inverse of $\frac{-15}{8}$ is $\frac{15}{8}$.

(iii) In standard form, we write $\frac{9}{-11}$ as $\frac{-9}{11}$.

Hence, its additive inverse is $\frac{9}{11}$.

(iv) We may write, $\frac{-6}{-7}=\frac{(-6)\times(-1)}{(-7)\times(-1)}=\frac{6}{7}$.

Hence, its additive inverse is $\frac{-6}{7}$.

EXAMPLE 2. *(i) Subtract $\frac{3}{4}$ from $\frac{2}{3}$.* *(ii) Subtract $\frac{-5}{7}$ from $\frac{-2}{5}$.*

Solution (i) $\left(\frac{2}{3}-\frac{3}{4}\right)=\frac{2}{3}+\left(\text{additive inverse of } \frac{3}{4}\right)$

$$=\left(\frac{2}{3}+\frac{-3}{4}\right)=\frac{\{8+(-9)\}}{12}=\frac{-1}{12}.$$

(ii) $\left\{\frac{-2}{5}-\left(\frac{-5}{7}\right)\right\}=\frac{-2}{5}+\left(\text{additive inverse of } \frac{-5}{7}\right)$

$$=\left(\frac{-2}{5}+\frac{5}{7}\right) \quad \left[\because \text{ additive inverse of } \frac{-5}{7} \text{ is } \frac{5}{7}\right]$$

$$=\frac{(-14+25)}{35}=\frac{11}{35}.$$

EXAMPLE 3. *The sum of two rational numbers is –5. If one of them is $\frac{-13}{6}$, find the other.*

Solution Let the other number be x. Then,

$$x+\left(\frac{-13}{6}\right)=-5 \Rightarrow x=-5+\left(\text{additive inverse of } \frac{-13}{6}\right)$$

$$\Rightarrow x=\left(-5+\frac{13}{6}\right)=\left(\frac{-5}{1}+\frac{13}{6}\right)=\frac{(-30+13)}{6}$$

$$\Rightarrow x=\frac{-17}{6}.$$

Hence, the required number is $\frac{-17}{6}$.

EXAMPLE 4. *What number should be added to $\frac{-7}{8}$ to get $\frac{4}{9}$?*

Solution Let the required number to be added be x. Then,

$$\frac{-7}{8}+x=\frac{4}{9} \Rightarrow x=\frac{4}{9}+\left(\text{additive inverse of } \frac{-7}{8}\right)$$

$$\Rightarrow x=\left(\frac{4}{9}+\frac{7}{8}\right)=\frac{(32+63)}{72}=\frac{95}{72}.$$

Hence, the required number is $\frac{95}{72}$.

EXAMPLE 5. *Evaluate* $\frac{3}{5}+\frac{7}{3}+\frac{-11}{5}+\frac{-2}{3}$.

Solution Using the commutative and associative laws, it follows that we may arrange the terms in any manner suitably. Using this rearrangement property, we have:

$$\frac{3}{5}+\frac{7}{3}+\frac{-11}{5}+\frac{-2}{3}=\left(\frac{3}{5}+\frac{-11}{5}\right)+\left(\frac{7}{3}+\frac{-2}{3}\right)$$

$$=\frac{\{3+(-11)\}}{5}+\frac{\{7+(-2)\}}{3}=\frac{-8}{5}+\frac{5}{3}$$

$$=\frac{(-24+25)}{15}=\frac{1}{15}.$$

EXAMPLE 6. *Simplify:* $\left(\frac{4}{7}+\frac{-8}{9}+\frac{-5}{21}+\frac{1}{3}\right)$.

Solution Using the rearrangement property, we have:

$$\frac{4}{7}+\frac{-8}{9}+\frac{-5}{21}+\frac{1}{3}=\left(\frac{4}{7}+\frac{-5}{21}\right)+\left(\frac{-8}{9}+\frac{1}{3}\right)$$

$$=\frac{\{12+(-5)\}}{21}+\frac{\{-8+3\}}{9}$$

$$=\left(\frac{7}{21}+\frac{-5}{9}\right)=\frac{\{21+(-35)\}}{63}=\frac{-14}{63}=\frac{-2}{9}.$$

EXAMPLE 7. *What should be subtracted from $\frac{-5}{7}$ to get -1?*

Solution Let the required number be x. Then,

$$\frac{-5}{7}-x=-1 \Rightarrow \frac{-5}{7}=x-1$$

$$\Rightarrow x=\left(\frac{-5}{7}+1\right)=\frac{(-5+7)}{7}=\frac{2}{7}.$$

Hence, the required number is $\frac{2}{7}$.

EXERCISE 1C

1. Add the following rational numbers:

(i) $\frac{-2}{5}$ and $\frac{4}{5}$ (ii) $\frac{-6}{11}$ and $\frac{-4}{11}$ (iii) $\frac{-11}{8}$ and $\frac{5}{8}$

(iv) $\frac{-7}{3}$ and $\frac{1}{3}$ (v) $\frac{5}{6}$ and $\frac{-1}{6}$ (vi) $\frac{-17}{15}$ and $\frac{-1}{15}$

2. Add the following rational numbers:

(i) $\frac{3}{4}$ and $\frac{-3}{5}$ (ii) $\frac{5}{8}$ and $\frac{-7}{12}$ (iii) $\frac{-8}{9}$ and $\frac{11}{6}$

(iv) $\frac{-5}{16}$ and $\frac{7}{24}$ (v) $\frac{7}{-18}$ and $\frac{8}{27}$ (vi) $\frac{1}{-12}$ and $\frac{2}{-15}$

(vii) -1 and $\frac{3}{4}$ (viii) 2 and $\frac{-5}{4}$ (ix) 0 and $\frac{-2}{5}$

3. Verify the following:

(i) $\frac{-12}{5}+\frac{2}{7}=\frac{2}{7}+\frac{-12}{5}$ (ii) $\frac{-5}{8}+\frac{-9}{13}=\frac{-9}{13}+\frac{-5}{8}$

(iii) $3+\frac{-7}{12}=\frac{-7}{12}+3$ (iv) $\frac{2}{-7}+\frac{12}{-35}=\frac{12}{-35}+\frac{2}{-7}$

4. Verify the following:

(i) $\left(\frac{3}{4}+\frac{-2}{5}\right)+\frac{-7}{10}=\frac{3}{4}+\left(\frac{-2}{5}+\frac{-7}{10}\right)$ (ii) $\left(\frac{-7}{11}+\frac{2}{-5}\right)+\frac{-13}{22}=\frac{-7}{11}+\left(\frac{2}{-5}+\frac{-13}{22}\right)$

(iii) $-1+\left(\frac{-2}{3}+\frac{-3}{4}\right)=\left(-1+\frac{-2}{3}\right)+\frac{-3}{4}$

5. Fill in the blanks.

(i) $\left(\frac{-3}{17}\right)+\left(\frac{-12}{5}\right)=\left(\frac{-12}{5}\right)+(\ldots\ldots)$ (ii) $-9+\frac{-21}{8}=(\ldots\ldots)+(-9)$

(iii) $\left(\frac{-8}{13}+\frac{3}{7}\right)+\left(\frac{-13}{4}\right)=(......)+\left[\frac{3}{7}+\left(\frac{-13}{4}\right)\right]$

(iv) $-12+\left(\frac{7}{12}+\frac{-9}{11}\right)=\left(-12+\frac{7}{12}\right)+(......)$

(v) $\frac{19}{-5}+\left(\frac{-3}{11}+\frac{-7}{8}\right)=\left\{\frac{19}{-5}+(......)\right\}+\frac{-7}{8}$

(vi) $\frac{-16}{7}+......=......+\frac{-16}{7}=\frac{-16}{7}$

6. Find the additive inverse of each of the following:

(i) $\frac{1}{3}$ (ii) $\frac{23}{9}$ (iii) -18 (iv) $\frac{-17}{8}$ (v) $\frac{15}{-4}$

(vi) $\frac{-16}{-5}$ (vii) $\frac{-3}{11}$ (viii) 0 (ix) $\frac{19}{-6}$ (x) $\frac{-8}{-7}$

7. Subtract:

(i) $\frac{3}{4}$ from $\frac{1}{3}$ (ii) $\frac{-5}{6}$ from $\frac{1}{3}$ (iii) $\frac{-8}{9}$ from $\frac{-3}{5}$ (iv) $\frac{-9}{7}$ from -1

(v) $\frac{-18}{11}$ from 1 (vi) $\frac{-13}{9}$ from 0 (vii) $\frac{-32}{13}$ from $\frac{-6}{5}$ (viii) -7 from $\frac{-4}{7}$

8. Using the rearrangement property find the sum:

(i) $\frac{4}{3}+\frac{3}{5}+\frac{-2}{3}+\frac{-11}{5}$ (ii) $\frac{-8}{3}+\frac{-1}{4}+\frac{-11}{6}+\frac{3}{8}$

(iii) $\frac{-13}{20}+\frac{11}{14}+\frac{-5}{7}+\frac{7}{10}$ (iv) $\frac{-6}{7}+\frac{-5}{6}+\frac{-4}{9}+\frac{-15}{7}$

9. The sum of two rational numbers is -2. If one of the numbers is $\frac{-14}{5}$, find the other.

10. The sum of two rational numbers is $\frac{-1}{2}$. If one of the numbers is $\frac{5}{6}$, find the other.

11. What number should be added to $\frac{-5}{8}$ so as to get $\frac{-3}{2}$?

12. What number should be added to -1 so as to get $\frac{5}{7}$?

13. What number should be subtracted from $\frac{-2}{3}$ to get $\frac{-1}{6}$?

14. (i) Which rational number is its own additive inverse?

(ii) Is the difference of two rational numbers a rational number?

(iii) Is addition commutative on rational numbers?

(iv) Is addition associative on rational numbers?

(v) Is subtraction commutative on rational numbers?

(vi) Is subtraction associative on rational numbers?

(vii) What is the negative of a negative rational number?

MULTIPLICATION OF RATIONAL NUMBERS

For any two rationals $\frac{a}{b}$ and $\frac{c}{d}$, we define:

$$\left(\frac{a}{b} \times \frac{c}{d}\right) = \frac{(a \times c)}{(b \times d)}.$$

SOLVED EXAMPLES

EXAMPLE 1. *Find each of the following products:*

(i) $\frac{2}{3} \times \frac{-5}{7}$ (ii) $\frac{-7}{8} \times \frac{3}{5}$ (iii) $\frac{-15}{4} \times \frac{-3}{8}$

Solution We have:

(i) $\frac{2}{3} \times \frac{-5}{7} = \frac{2 \times (-5)}{3 \times 7} = \frac{-10}{21}.$

(ii) $\frac{-7}{8} \times \frac{3}{5} = \frac{(-7) \times 3}{8 \times 5} = \frac{-21}{40}.$

(iii) $\frac{-15}{4} \times \frac{-3}{8} = \frac{(-15) \times (-3)}{4 \times 8} = \frac{45}{32}.$

EXAMPLE 2. *Find each of the following products:*

(i) $\frac{-3}{7} \times \frac{14}{5}$ (ii) $\frac{13}{6} \times \frac{-18}{91}$ (iii) $\frac{-11}{9} \times \frac{-51}{44}$

Solution We have:

(i) $\frac{-3}{7} \times \frac{14}{5} = \frac{(-3) \times \cancel{14}^{2}}{{}_{1}\cancel{7} \times 5} = \frac{-6}{5}.$

(ii) $\frac{13}{6} \times \frac{-18}{91} = \frac{13 \times (-18)}{6 \times 91} = \frac{-(\cancel{13}^{1} \times \cancel{18}^{3})}{(\cancel{6}_{1} \times \cancel{91}_{7})} = \frac{-3}{7}.$

(iii) $\frac{-11}{9} \times \frac{-51}{44} = \frac{(-11) \times (-51)}{9 \times 44} = \frac{\cancel{11}^{1} \times \cancel{51}^{17}}{\cancel{9}_{3} \times \cancel{44}_{4}} = \frac{17}{12}.$

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

Property 1 (Closure Property): *The product of two rational numbers is always a rational number.*

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\left(\frac{a}{b} \times \frac{c}{d}\right)$ is also a rational number.

EXAMPLES (i) Consider the rational numbers $\frac{1}{2}$ and $\frac{5}{7}$. Then,

$\left(\frac{1}{2} \times \frac{5}{7}\right) = \frac{(1 \times 5)}{(2 \times 7)} = \frac{5}{14}$, which is a rational number.

(ii) Consider the rational numbers $\frac{-3}{7}$ and $\frac{5}{14}$. Then,

$\left(\frac{-3}{7} \times \frac{5}{14}\right) = \frac{(-3) \times 5}{7 \times 14} = \frac{-15}{98}$, which is a rational number.

(iii) Consider the rational numbers $\frac{-4}{5}$ and $\frac{-7}{3}$. Then,

$\left(\frac{-4}{5} \times \frac{-7}{3}\right) = \frac{(-4) \times (-7)}{5 \times 3} = \frac{28}{15}$, which is a rational number.

Property 2 (Commutative Law): *Two rational numbers can be multiplied in any order.*

Thus, for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right).$$

EXAMPLES (i) Let us consider the rational numbers $\frac{3}{4}$ and $\frac{5}{7}$. Then,

$$\left(\frac{3}{4}\times\frac{5}{7}\right)=\frac{(3\times 5)}{(4\times 7)}=\frac{15}{28} \text{ and } \left(\frac{5}{7}\times\frac{3}{4}\right)=\frac{(5\times 3)}{(7\times 4)}=\frac{15}{28}.$$

$$\therefore \left(\frac{3}{4}\times\frac{5}{7}\right)=\left(\frac{5}{7}\times\frac{3}{4}\right).$$

(ii) Let us consider the rational numbers $\frac{-2}{5}$ and $\frac{6}{7}$. Then,

$$\left(\frac{-2}{5}\times\frac{6}{7}\right)=\frac{(-2)\times 6}{5\times 7}=\frac{-12}{35} \text{ and } \left(\frac{6}{7}\times\frac{-2}{5}\right)=\frac{6\times(-2)}{7\times 5}=\frac{-12}{35}.$$

$$\therefore \left(\frac{-2}{5}\times\frac{6}{7}\right)=\left(\frac{6}{7}\times\frac{-2}{5}\right).$$

(iii) Let us consider the rational numbers $\frac{-2}{3}$ and $\frac{-5}{7}$. Then,

$$\left(\frac{-2}{3}\right)\times\left(\frac{-5}{7}\right)=\frac{(-2)\times(-5)}{3\times 7}=\frac{10}{21} \text{ and } \left(\frac{-5}{7}\right)\times\left(\frac{-2}{3}\right)=\frac{(-5)\times(-2)}{7\times 3}=\frac{10}{21}.$$

$$\therefore \left(\frac{-2}{3}\right)\times\left(\frac{-5}{7}\right)=\left(\frac{-5}{7}\right)\times\left(\frac{-2}{3}\right).$$

Property 3 (Associative Law): *While multiplying three or more rational numbers, they can be grouped in any order.*

Thus, for any rationals $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right).$$

EXAMPLE Consider the rationals $\frac{-5}{2}$, $\frac{-7}{4}$ and $\frac{1}{3}$. We have

$$\left(\frac{-5}{2}\times\frac{-7}{4}\right)\times\frac{1}{3}=\left\{\frac{(-5)\times(-7)}{2\times 4}\times\frac{1}{3}\right\}=\left(\frac{35}{8}\times\frac{1}{3}\right)=\frac{(35\times 1)}{(8\times 3)}=\frac{35}{24}$$

$$\text{and } \frac{-5}{2}\times\left(\frac{-7}{4}\times\frac{1}{3}\right)=\frac{-5}{2}\times\frac{(-7)\times 1}{4\times 3}=\left(\frac{-5}{2}\times\frac{-7}{12}\right)=\frac{(-5)\times(-7)}{(2\times 12)}=\frac{35}{24}.$$

$$\therefore \left(\frac{-5}{2}\times\frac{-7}{4}\right)\times\frac{1}{3}=\frac{-5}{2}\times\left(\frac{-7}{4}\times\frac{1}{3}\right).$$

Property 4 (Existence of Multiplicative Identity):

For any rational number $\frac{a}{b}$, *we have* $\left(\frac{a}{b}\times 1\right)=\left(1\times\frac{a}{b}\right)=\frac{a}{b}$.

1 is called the ***multiplicative identity*** for rationals.

EXAMPLES (i) Consider the rational number $\frac{3}{4}$. Then, we have

$$\left(\frac{3}{4}\times 1\right)=\left(\frac{3}{4}\times\frac{1}{1}\right)=\frac{(3\times 1)}{(4\times 1)}=\frac{3}{4} \text{ and } \left(1\times\frac{3}{4}\right)=\left(\frac{1}{1}\times\frac{3}{4}\right)=\frac{(1\times 3)}{(1\times 4)}=\frac{3}{4}.$$

$\therefore \quad \left(\frac{3}{4}\times 1\right)=\left(1\times\frac{3}{4}\right)=\frac{3}{4}.$

(ii) Consider the rational number $\frac{-9}{13}$. Then, we have

$$\left(\frac{-9}{13}\times 1\right)=\left(\frac{-9}{13}\times\frac{1}{1}\right)=\frac{(-9)\times 1}{13\times 1}=\frac{-9}{13} \text{ and } \left(1\times\frac{-9}{13}\right)=\left(\frac{1}{1}\times\frac{-9}{13}\right)=\frac{1\times(-9)}{1\times 13}=\frac{-9}{13}.$$

$\therefore \quad \left(\frac{-9}{13}\times 1\right)=\left(1\times\frac{-9}{13}\right)=\frac{-9}{13}.$

Property 5 (Existence of Multiplicative Inverse): *Every nonzero rational number* $\frac{a}{b}$ *has its multiplicative inverse* $\frac{b}{a}$.

Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1.$

$\frac{b}{a}$ *is called the* ***reciprocal*** *of* $\frac{a}{b}$.

Clearly, *zero has no reciprocal.*

Reciprocal of 1 *is* 1 *and the reciprocal of* (−1) *is* (−1).

EXAMPLES (i) Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1.$

(ii) Reciprocal of $\frac{-8}{9}$ is $\frac{-9}{8}$, since $\left(\frac{-8}{9}\times\frac{-9}{8}\right)=\left(\frac{-9}{8}\times\frac{-8}{9}\right)=1.$

(iii) Reciprocal of −3 is $\frac{-1}{3}$, since

$$\left(-3\times\frac{-1}{3}\right)=\left(\frac{-3}{1}\times\frac{-1}{3}\right)=\frac{(-3)\times(-1)}{1\times 3}=\frac{3}{3}=1 \text{ and } \left(\frac{-1}{3}\times -3\right)=\left(\frac{-1}{3}\times\frac{-3}{1}\right)=\frac{(-1)\times(-3)}{3\times 1}=1.$$

REMARK We denote the reciprocal of $\frac{a}{b}$ by $\left(\frac{a}{b}\right)^{-1}$.

Clearly, $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}.$

Property 6 (Distributive Law of Multiplication Over Addition): *For any three rational numbers* $\frac{a}{b}$, $\frac{c}{d}$ *and* $\frac{e}{f}$, *we have*

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right).$$

EXAMPLE Consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$. We have

$$\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\right)\times\left\{\frac{4+(-5)}{6}\right\}=\left(\frac{-3}{4}\right)\times\left(\frac{-1}{6}\right)=\frac{(-3)\times(-1)}{4\times 6}=\frac{3}{24}=\frac{1}{8}.$$

Again, $\left(\frac{-3}{4}\right)\times\frac{2}{3}=\frac{(-3)\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$ and $\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)=\frac{(-3)\times(-5)}{4\times 6}=\frac{15}{24}=\frac{5}{8}.$

$$\therefore \left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}=\left(\frac{-1}{2}+\frac{5}{8}\right)=\frac{(-4+5)}{8}=\frac{1}{8}.$$

Hence, $\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}.$

Property 7 (Multiplicative Property of 0): *Every rational number multiplied with 0 gives 0.*

Thus, for any rational number $\frac{a}{b}$, we have: $\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\times\mathbf{0}\right)=\left(\mathbf{0}\times\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)=\mathbf{0}.$

EXAMPLES (i) $\left(\frac{5}{18}\times 0\right)=\left(\frac{5}{18}\times\frac{0}{1}\right)=\frac{(5\times 0)}{(18\times 1)}=\frac{0}{18}=0.$ Similarly, $\left(0\times\frac{5}{18}\right)=0.$

(ii) $\left(\frac{-12}{17}\times 0\right)=\left(\frac{-12}{17}\times\frac{0}{1}\right)=\frac{(-12)\times 0}{17\times 1}=\frac{0}{17}=0.$ Similarly, $\left(0\times\frac{-12}{17}\right)=0.$

SOLVED EXAMPLES

EXAMPLE 1. *Find the reciprocal of each of the following:*

(i) 12 *(ii)* −8 *(iii)* $\frac{5}{16}$ *(iv)* $\frac{-14}{17}$

Solution (i) Reciprocal of 12 is $\frac{1}{12}$.

(ii) Reciprocal of −8 is $\frac{1}{-8}$, i.e., $\frac{-1}{8}$.

(iii) Reciprocal of $\frac{5}{16}$ is $\frac{16}{5}$.

(iv) Reciprocal of $\frac{-14}{17}$ is $\frac{17}{-14}$, i.e., $\frac{-17}{14}$.

EXAMPLE 2. *Verify that:*

(i) $\left(\frac{-3}{16}\times\frac{8}{15}\right)=\left(\frac{8}{15}\times\frac{-3}{16}\right)$ *(ii)* $\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$

(iii) $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$

Solution (i) LHS $=\left(\frac{-3}{16}\times\frac{8}{15}\right)=\frac{(-3)\times 8}{16\times 15}=\frac{-24}{240}=\frac{-1}{10}.$

RHS $=\left(\frac{8}{15}\times\frac{-3}{16}\right)=\frac{8\times(-3)}{15\times 16}=\frac{-24}{240}=\frac{-1}{10}.$

∴ LHS = RHS.

Hence, $\left(\frac{-3}{16}\times\frac{8}{15}\right)=\left(\frac{8}{15}\times\frac{-3}{16}\right).$

(ii) LHS $=\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\frac{2}{3}\times\frac{6\times(-14)}{7\times 15}=\frac{2}{3}\times\frac{-84}{105}$

$=\frac{2}{3}\times\frac{-4}{5}=\frac{2\times(-4)}{3\times 5}=\frac{-8}{15}.$

RHS $=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}=\frac{(2\times 6)}{(3\times 7)}\times\frac{-14}{15}=\frac{12}{21}\times\frac{-14}{15}$

$=\frac{4}{7}\times\frac{-14}{15}=\frac{4\times(-14)}{(7\times 15)}=\frac{-56}{105}=\frac{-8}{15}.$

∴ LHS = RHS.

Hence, $\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}.$

(iii) LHS $= \frac{5}{6} \times \left(\frac{-4}{5} + \frac{-7}{10}\right) = \frac{5}{6} \times \left[\frac{(-8)+(-7)}{10}\right] = \frac{5}{6} \times \frac{-15}{10}$

$= \frac{5}{6} \times \frac{-3}{2} = \frac{5 \times (-3)}{6 \times 2} = \frac{-15}{12} = \frac{-5}{4}.$

RHS $= \left(\frac{5}{6} \times \frac{-4}{5}\right) + \left(\frac{5}{6} \times \frac{-7}{10}\right)$

$= \frac{5 \times (-4)}{6 \times 5} + \frac{5 \times (-7)}{6 \times 10} = \frac{-20}{30} + \frac{-35}{60} = \frac{-2}{3} + \frac{-7}{12}$

$= \frac{(-8)+(-7)}{12} = \frac{-15}{12} = \frac{-5}{4}.$

$\therefore$ LHS = RHS.

Hence, $\frac{5}{6} \times \left(\frac{-4}{5} + \frac{-7}{10}\right) = \left(\frac{5}{6} \times \frac{-4}{5}\right) + \left(\frac{5}{6} \times \frac{-7}{10}\right).$

EXERCISE 1D

1. Find each of the following products:

(i) $\frac{3}{5} \times \frac{-7}{8}$ (ii) $\frac{-9}{2} \times \frac{5}{4}$ (iii) $\frac{-6}{11} \times \frac{-5}{3}$

(iv) $\frac{-2}{3} \times \frac{6}{7}$ (v) $\frac{-12}{5} \times \frac{10}{-3}$ (vi) $\frac{25}{-9} \times \frac{3}{-10}$

(vii) $\frac{5}{-18} \times \frac{-9}{20}$ (viii) $\frac{-13}{15} \times \frac{-25}{26}$ (ix) $\frac{16}{-21} \times \frac{14}{5}$

(x) $\frac{-7}{6} \times 24$ (xi) $\frac{7}{24} \times (-48)$ (xii) $\frac{-13}{5} \times (-10)$

2. Verify each of the following:

(i) $\frac{3}{7} \times \frac{-5}{9} = \frac{-5}{9} \times \frac{3}{7}$ (ii) $\frac{-8}{7} \times \frac{13}{9} = \frac{13}{9} \times \frac{-8}{7}$

(iii) $\frac{-12}{5} \times \frac{7}{-36} = \frac{7}{-36} \times \frac{-12}{5}$ (iv) $-8 \times \frac{-13}{12} = \frac{-13}{12} \times (-8)$

3. Verify each of the following:

(i) $\left(\frac{5}{7} \times \frac{12}{13}\right) \times \frac{7}{18} = \frac{5}{7} \times \left(\frac{12}{13} \times \frac{7}{18}\right)$ (ii) $\frac{-13}{24} \times \left(\frac{-12}{5} \times \frac{35}{36}\right) = \left(\frac{-13}{24} \times \frac{-12}{5}\right) \times \frac{35}{36}$

(iii) $\left(\frac{-9}{5} \times \frac{-10}{3}\right) \times \frac{21}{-4} = \frac{-9}{5} \times \left(\frac{-10}{3} \times \frac{21}{-4}\right)$

4. Fill in the blanks:

(i) $\frac{-23}{17} \times \frac{18}{35} = \frac{18}{35} \times (\ldots\ldots)$ (ii) $-38 \times \frac{-7}{19} = \frac{-7}{19} \times (\ldots\ldots)$

(iii) $\left(\frac{15}{7} \times \frac{-21}{10}\right) \times \frac{-5}{6} = (\ldots\ldots) \times \left(\frac{-21}{10} \times \frac{-5}{6}\right)$ (iv) $\frac{-12}{5} \times \left(\frac{4}{15} \times \frac{25}{-16}\right) = \left(\frac{-12}{5} \times \frac{4}{15}\right) \times (\ldots\ldots)$

5. Find the multiplicative inverse (i.e., reciprocal) of:

(i) $\frac{13}{25}$ (ii) $\frac{-17}{12}$ (iii) $\frac{-7}{24}$ (iv) 18 (v) -16

(vi) $\frac{-3}{-5}$ (vii) -1 (viii) $\frac{0}{2}$ (ix) $\frac{2}{-5}$ (x) $\frac{-1}{8}$

6. Find the value of:

(i) $\left(\frac{5}{8}\right)^{-1}$ (ii) $\left(\frac{-4}{9}\right)^{-1}$ (iii) $(-7)^{-1}$ (iv) $\left(\frac{1}{-3}\right)^{-1}$

7. Verify the following:

(i) $\frac{3}{7}\times\left(\frac{5}{6}+\frac{12}{13}\right)=\left(\frac{3}{7}\times\frac{5}{6}\right)+\left(\frac{3}{7}\times\frac{12}{13}\right)$

(ii) $\frac{-15}{4}\times\left(\frac{3}{7}+\frac{-12}{5}\right)=\left(\frac{-15}{4}\times\frac{3}{7}\right)+\left(\frac{-15}{4}\times\frac{-12}{5}\right)$

(iii) $\left(\frac{-8}{3}+\frac{-13}{12}\right)\times\frac{5}{6}=\left(\frac{-8}{3}\times\frac{5}{6}\right)+\left(\frac{-13}{12}\times\frac{5}{6}\right)$

(iv) $\frac{-16}{7}\times\left(\frac{-8}{9}+\frac{-7}{6}\right)=\left(\frac{-16}{7}\times\frac{-8}{9}\right)+\left(\frac{-16}{7}\times\frac{-7}{6}\right)$

8. Name the property of multiplication illustrated by each of the following statements:

(i) $\frac{-15}{8}\times\frac{-12}{7}=\frac{-12}{7}\times\frac{-15}{8}$

(ii) $\left(\frac{-2}{3}\times\frac{7}{9}\right)\times\frac{-9}{5}=\frac{-2}{3}\times\left(\frac{7}{9}\times\frac{-9}{5}\right)$

(iii) $\frac{-3}{4}\times\left(\frac{-5}{6}+\frac{7}{8}\right)=\left(\frac{-3}{4}\times\frac{-5}{6}\right)+\left(\frac{-3}{4}\times\frac{7}{8}\right)$

(iv) $\frac{-16}{9}\times 1=1\times\frac{-16}{9}=\frac{-16}{9}$

(v) $\frac{-11}{15}\times\frac{15}{-11}=\frac{15}{-11}\times\frac{-11}{15}=1$

(vi) $\frac{-7}{5}\times 0=0$

9. Fill in the blanks:

(i) The product of a rational number and its reciprocal is ....
(ii) Zero has ...... reciprocal.
(iii) The numbers ...... and ...... are their own reciprocals.
(iv) Zero is ...... the reciprocal of any number.
(v) The reciprocal of a, where $a \neq 0$, is ....
(vi) The reciprocal of $\frac{1}{a}$, where $a \neq 0$, is ....
(vii) The reciprocal of a positive rational number is ....
(viii) The reciprocal of a negative rational number is ....

DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d}\neq 0$, we define, $\left(\frac{a}{b}\div\frac{c}{d}\right)=\left(\frac{a}{b}\times\frac{d}{c}\right)$.

When $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is called the ***dividend;*** $\frac{c}{d}$ is called the ***divisor*** and the result is known as ***quotient.***

SOLVED EXAMPLES

EXAMPLE 1. *Divide:*

(i) $\frac{9}{16}$ *by* $\frac{5}{8}$ (ii) $\frac{-6}{25}$ *by* $\frac{3}{5}$ (iii) $\frac{11}{24}$ *by* $\frac{-5}{8}$ (iv) $\frac{-9}{40}$ *by* $\frac{-3}{8}$

Solution We have:

(i) $\frac{9}{16} \div \frac{5}{8} = \frac{9}{16} \times \frac{8}{5} = \frac{9 \times 8}{16 \times 5} = \frac{72}{80} = \frac{9}{10}$.

(ii) $\frac{-6}{25} \div \frac{3}{5} = \frac{-6}{25} \times \frac{5}{3} = \frac{(-6) \times 5}{25 \times 3} = \frac{-30}{75} = \frac{-2}{5}$.

(iii) $\frac{11}{24} \div \frac{-5}{8} = \frac{11}{24} \times \frac{8}{-5} = \frac{11 \times 8}{24 \times (-5)} = \frac{88}{-120} = \frac{-11}{15}$.

(iv) $\frac{-9}{40} \div \frac{-3}{8} = \frac{-9}{40} \times \frac{8}{-3} = \frac{(-9) \times 8}{40 \times (-3)} = \frac{-72}{-120} = \frac{3}{5}$.

EXAMPLE 2. *The product of two numbers is* $\frac{-28}{27}$. *If one of the numbers is* $\frac{-4}{9}$, *find the other.*

Solution Let the other number be x. Then,

$$x \times \frac{-4}{9} = \frac{-28}{27}$$

$$\Rightarrow \quad x = \frac{-28}{27} \div \frac{-4}{9} = \frac{-28}{27} \times \frac{9}{-4} = \frac{(-28) \times 9}{27 \times (-4)} = \frac{-(28 \times 9)}{-(27 \times 4)}$$

$$\Rightarrow \quad x = \frac{\cancel{28}^{7} \times \cancel{9}^{1}}{\cancel{27}_{3} \times \cancel{4}_{1}} = \frac{7}{3}.$$

Hence, the other number is $\frac{7}{3}$.

EXAMPLE 3. *Fill in the blanks:* $\frac{27}{16} \div (......) = \frac{-15}{8}$.

Solution Let $\frac{27}{16} \div \left(\frac{a}{b}\right) = \frac{-15}{8}$. Then,

$$\frac{27}{16} \times \frac{b}{a} = \frac{-15}{8} \quad \Rightarrow \quad \frac{b}{a} = \frac{-15}{8} \times \frac{16}{27} = \frac{-10}{9}$$

$$\Rightarrow \quad \frac{a}{b} = \frac{9}{-10} = \frac{-9}{10}.$$

Hence, the missing number is $\frac{-9}{10}$.

PROPERTIES OF DIVISION

Property 1 (Closure Property): *If* $\frac{a}{b}$ *and* $\frac{c}{d}$ *are any two rational numbers such that* $\frac{c}{d} \neq 0$ *then* $\left(\frac{a}{b} \div \frac{c}{d}\right)$ *is also a rational number.*

Property 2 (Property of 1): *For every rational number* $\frac{a}{b}$, *we have:*

$$\left(\frac{a}{b} \div 1\right) = \frac{a}{b}.$$

Property 3: *For every nonzero rational number* $\frac{a}{b}$, *we have:*

$$\left(\frac{a}{b} \div \frac{a}{b}\right) = 1.$$

EXERCISE 1E

1. Simplify:

   (i) $\frac{4}{9} \div \frac{-5}{12}$ (ii) $-8 \div \frac{-7}{16}$ (iii) $\frac{-12}{7} \div (-18)$

   (iv) $\frac{-1}{10} \div \frac{-8}{5}$ (v) $\frac{-16}{35} \div \frac{-15}{14}$ (vi) $\frac{-65}{14} \div \frac{13}{7}$

2. Verify whether the given statement is true or false:

   (i) $\frac{13}{5} \div \frac{26}{10} = \frac{26}{10} \div \frac{13}{5}$ (ii) $-9 \div \frac{3}{4} = \frac{3}{4} \div (-9)$

   (iii) $\frac{-8}{9} \div \frac{-4}{3} = \frac{-4}{3} \div \frac{-8}{9}$ (iv) $\frac{-7}{24} \div \frac{3}{-16} = \frac{3}{-16} \div \frac{-7}{24}$

3. Verify whether the given statement is true or false:

   (i) $\left(\frac{5}{9} \div \frac{1}{3}\right) \div \frac{5}{2} = \frac{5}{9} \div \left(\frac{1}{3} \div \frac{5}{2}\right)$ (ii) $\left\{(-16) \div \frac{6}{5}\right\} \div \frac{-9}{10} = (-16) \div \left\{\frac{6}{5} \div \frac{-9}{10}\right\}$

   (iii) $\left(\frac{-3}{5} \div \frac{-12}{35}\right) \div \frac{1}{14} = \frac{-3}{5} \div \left(\frac{-12}{35} \div \frac{1}{14}\right)$

4. The product of two rational numbers is –9. If one of the numbers is –12, find the other.

5. The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.

6. By what rational number should we multiply $\frac{-15}{56}$ to get $\frac{-5}{7}$?

7. By what rational number should $\frac{-8}{39}$ be multiplied to obtain $\frac{1}{26}$?

8. By what number should $\frac{-33}{8}$ be divided to get $\frac{-11}{2}$?

9. Divide the sum of $\frac{13}{5}$ and $\frac{-12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{1}{-2}$.

10. Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.

11. Fill in the blanks:

    (i) $\frac{9}{8} \div (......) = \frac{-3}{2}$ (ii) $(......) \div \left(\frac{-7}{5}\right) = \frac{10}{19}$

    (iii) $(......) \div (-3) = \frac{-4}{15}$ (iv) $(-12) \div (......) = \frac{-6}{5}$

12. (i) Are rational numbers always closed under division?

    (ii) Are rational numbers always commutative under division?

    (iii) Are rational numbers always associative under division?

    (iv) Can we divide 1 by 0?

An Important Result:

If x and y be two rational numbers such that $x < y$ then $\frac{1}{2}(x+y)$ is a rational number between x and y.

EXAMPLE 1. *Find a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.*

Solution Required number $= \frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)$

$$= \frac{1}{2}\left(\frac{2+3}{6}\right) = \left(\frac{1}{2}\times\frac{5}{6}\right) = \frac{5}{12}.$$

Hence, $\frac{5}{12}$ is a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

EXAMPLE 2. *Find three rational numbers lying between 3 and 4.*

Solution A rational number between 3 and 4 is $\frac{1}{2}(3+4) = \frac{7}{2}$.

Then, $3 < \frac{7}{2} < 4$.

A rational number between 3 and $\frac{7}{2} = \frac{1}{2}\left(3+\frac{7}{2}\right) = \frac{1}{2}\left(\frac{3}{1}+\frac{7}{2}\right)$

$$= \frac{1}{2}\left(\frac{6+7}{2}\right) = \left(\frac{1}{2}\times\frac{13}{2}\right) = \frac{13}{4}.$$

A rational number between $\frac{7}{2}$ and $4 = \frac{1}{2}\left(\frac{7}{2}+4\right) = \frac{1}{2}\left(\frac{7}{2}+\frac{4}{1}\right)$

$$= \frac{1}{2}\left(\frac{7+8}{2}\right) = \left(\frac{1}{2}\times\frac{15}{2}\right) = \frac{15}{4}.$$

$\therefore \quad 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4.$

Hence, the required numbers are $\frac{13}{4}, \frac{7}{2}$ and $\frac{15}{4}$.

ALTERNATIVE METHOD OF FINDING LARGE NUMBER OF RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

EXAMPLE 3. *Find 20 rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.*

Solution LCM of 6 and 8 is 24.

Now, $\frac{-5}{6} = \frac{-5\times4}{6\times4} = \frac{-20}{24}$ and $\frac{5}{8} = \frac{5\times3}{8\times3} = \frac{15}{24}$.

Rational numbers lying between $\frac{-5}{6}$ and $\frac{5}{8}$ are

$$\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \frac{-16}{24}, \ldots, \frac{-1}{24}, \frac{0}{24}, \frac{1}{24}, \frac{2}{24}, \frac{3}{24}, \ldots, \frac{14}{24}.$$

Out of these 20 may be taken.

EXAMPLE 4. *Find 15 rational numbers between −2 and 0.*

Solution We may write, $-2 = \frac{-20}{10}$ and $0 = \frac{0}{10}$.

Rational numbers lying between −2 and 0 are

$\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \frac{-14}{10}, \frac{-13}{10}, \frac{-12}{10}, \frac{-11}{10}, -1, \frac{-9}{10},$
$\frac{-8}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}, \frac{-4}{10}, \frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}.$

Out of these 15 may be taken.

EXAMPLE 5. *Write 9 rational numbers between 1 and 2.*

Solution We may write $1 = \frac{10}{10}$ and $2 = \frac{20}{10}$

∴ rational numbers between 1 and 2 are

$\frac{11}{10}, \frac{12}{10}, \frac{13}{10}, \frac{14}{10}, \frac{15}{10}, \frac{16}{10}, \frac{17}{10}, \frac{18}{10}, \frac{19}{10}.$

REMARK *Suppose we have to write 99 rational numbers between 1 and 2.*

Then, we may write, $1 = \frac{100}{100}$ and $2 = \frac{200}{100}$.

∴ rational numbers between 1 and 2 are

$\frac{101}{100}, \frac{102}{100}, \frac{103}{100}, \ldots, \frac{198}{100}, \frac{199}{100}.$

EXERCISE 1F

1. Find a rational number between $\frac{1}{4}$ and $\frac{1}{3}$.
2. Find a rational number between 2 and 3.
3. Find a rational number between $\frac{-1}{3}$ and $\frac{1}{2}$.
4. Find two rational numbers between –3 and –2.
5. Find three rational numbers between 4 and 5.
6. Find three rational numbers between $\frac{2}{3}$ and $\frac{3}{4}$.
7. Find 10 rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$.
8. Find 12 rational numbers between –1 and 2.

WORD PROBLEMS

EXERCISE 1G

1. From a rope 11 m long, two pieces of lengths $2\frac{3}{5}$ m and $3\frac{3}{10}$ m are cut off. What is the length of the remaining rope?
2. A drum full of rice weighs $40\frac{1}{6}$ kg. If the empty drum weighs $13\frac{3}{4}$ kg, find the weight of rice in the drum.
3. A basket contains three types of fruits weighing $19\frac{1}{3}$ kg in all. If $8\frac{1}{9}$ kg of these be apples, $3\frac{1}{6}$ kg be oranges and the rest pears, what is the weight of the pears in the basket?

4. On one day a rickshaw puller earned ₹ 160. Out of his earnings he spent ₹ $26\frac{3}{5}$ on tea and snacks, ₹ $50\frac{1}{2}$ on food and ₹ $16\frac{2}{5}$ on repairs of the rickshaw. How much did he save on that day?

5. Find the cost of $3\frac{2}{5}$ metres of cloth at ₹ $63\frac{3}{4}$ per metre.

6. A car is moving at an average speed of $60\frac{2}{5}$ km/hr. How much distance will it cover in $6\frac{1}{4}$ hours?

7. Find the area of a rectangular park which is $36\frac{3}{5}$ m long and $16\frac{2}{3}$ m broad.

8. Find the area of a square plot of land whose each side measures $8\frac{1}{2}$ metres.

9. One litre of petrol costs ₹ $63\frac{3}{4}$. What is the cost of 34 litres of petrol?

10. An aeroplane covers 1020 km in an hour. How much distance will it cover in $4\frac{1}{6}$ hours?

11. The cost of $3\frac{1}{2}$ metres of cloth is ₹ $166\frac{1}{4}$. What is the cost of one metre of cloth?

12. A cord of length $71\frac{1}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?

13. The area of a room is $65\frac{1}{4}$ m^2. If its breadth is $5\frac{7}{16}$ metres, what is its length?

14. The product of two fractions is $9\frac{3}{5}$. If one of the fractions is $9\frac{3}{7}$, find the other.

15. In a school, $\frac{5}{8}$ of the students are boys. If there are 240 girls, find the number of boys in the school.

16. After reading $\frac{7}{9}$ of a book, 40 pages are left. How many pages are there in the book?

17. Rita had ₹ 300. She spent $\frac{1}{3}$ of her money on notebooks and $\frac{1}{4}$ of the remainder on stationery items. How much money is left with her?

18. Amit earns ₹ 32000 per month. He spends $\frac{1}{4}$ of his income on food; $\frac{3}{10}$ of the remainder on house rent and $\frac{5}{21}$ of the remainder on the education of children. How much money is still left with him?

19. If $\frac{3}{5}$ of a number exceeds its $\frac{2}{7}$ by 44, find the number.

20. At a cricket test match $\frac{2}{7}$ of the spectators were in a covered place while 15000 were in open. Find the total number of spectators.

EXERCISE 1H

OBJECTIVE QUESTIONS

Tick (✓) the correct answer in each of the following:

1. $\left(\frac{-5}{16}+\frac{7}{12}\right)=?$

(a) $-\frac{7}{48}$ (b) $\frac{1}{24}$ (c) $\frac{13}{48}$ (d) $\frac{1}{3}$

2. $\left(\frac{8}{-15}+\frac{4}{-3}\right)=?$

(a) $\frac{28}{15}$ (b) $\frac{-28}{15}$ (c) $\frac{-4}{5}$ (d) $\frac{-4}{15}$

3. $\left(\frac{7}{-26}+\frac{16}{39}\right)=?$

(a) $\frac{11}{78}$ (b) $\frac{-11}{78}$ (c) $\frac{11}{39}$ (d) $\frac{-11}{39}$

4. $\left(3+\frac{5}{-7}\right)=?$

(a) $\frac{-16}{7}$ (b) $\frac{16}{7}$ (c) $\frac{-26}{7}$ (d) $\frac{-8}{7}$

5. $\left(\frac{31}{-4}+\frac{-5}{8}\right)=?$

(a) $\frac{67}{8}$ (b) $\frac{57}{8}$ (c) $\frac{-57}{8}$ (d) $\frac{-67}{8}$

6. What should be added to $\frac{7}{12}$ to get $\frac{-4}{15}$?

(a) $\frac{17}{20}$ (b) $\frac{-17}{20}$ (c) $\frac{7}{20}$ (d) $\frac{-7}{20}$

7. $\left(\frac{2}{3}+\frac{-4}{5}+\frac{7}{15}+\frac{-11}{20}\right)=?$

(a) $\frac{-1}{5}$ (b) $\frac{-4}{15}$ (c) $\frac{-13}{60}$ (d) $\frac{-7}{30}$

8. The sum of two numbers is $\frac{-4}{3}$. If one of the numbers is –5, what is the other?

(a) $\frac{-11}{3}$ (b) $\frac{11}{3}$ (c) $\frac{-19}{3}$ (d) $\frac{19}{3}$

9. What should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

(a) $\frac{-29}{21}$ (b) $\frac{29}{21}$ (c) $\frac{1}{21}$ (d) $\frac{-1}{21}$

10. What should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) $\frac{-5}{2}$

11. $\left(\frac{-3}{7}\right)^{-1}=?$

(a) $\frac{7}{3}$ (b) $\frac{-7}{3}$ (c) $\frac{3}{7}$ (d) none of these

12. The product of two rational numbers is $\frac{-28}{81}$. If one of the numbers is $\frac{14}{27}$ then the other one is

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

13. The product of two numbers is $\frac{-16}{35}$. If one of the numbers is $\frac{-15}{14}$, the other is

(a) $\frac{-2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{32}{75}$ (d) $\frac{-8}{3}$

14. What should be subtracted from $\frac{-3}{5}$ to get –2?

(a) $\frac{-7}{5}$ (b) $\frac{-13}{5}$ (c) $\frac{13}{5}$ (d) $\frac{7}{5}$

15. The sum of two rational numbers is –3. If one of them is $\frac{-10}{3}$ then the other one is

(a) $\frac{-13}{3}$ (b) $\frac{-19}{3}$ (c) $\frac{1}{3}$ (d) $\frac{13}{3}$

16. Which of the following numbers is in standard form?

(a) $\frac{-12}{26}$ (b) $\frac{-49}{71}$ (c) $\frac{-9}{16}$ (d) $\frac{28}{-105}$

17. $\left(\frac{-9}{16} \times \frac{8}{15}\right) = ?$

(a) $\frac{-3}{10}$ (b) $\frac{-4}{15}$ (c) $\frac{-9}{25}$ (d) $\frac{-2}{5}$

18. $\left(\frac{-5}{9} \div \frac{2}{3}\right) = ?$

(a) $\frac{-5}{2}$ (b) $\frac{-5}{6}$ (c) $\frac{-10}{27}$ (d) $\frac{-6}{5}$

19. $\frac{4}{9} \div ? = \frac{-8}{15}$

(a) $\frac{-32}{45}$ (b) $\frac{-8}{5}$ (c) $\frac{-9}{10}$ (d) $\frac{-5}{6}$

20. Additive inverse of $\frac{-5}{9}$ is

(a) $\frac{-9}{5}$ (b) 0 (c) $\frac{5}{9}$ (d) $\frac{9}{5}$

21. Reciprocal of $\frac{-3}{4}$ is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-4}{3}$ (d) 0

22. A rational number between $\frac{-2}{3}$ and $\frac{1}{4}$ is

(a) $\frac{5}{12}$ (b) $\frac{-5}{12}$ (c) $\frac{5}{24}$ (d) $\frac{-5}{24}$

23. The reciprocal of a negative rational number

(a) is a positive rational number
(b) is a negative rational number
(c) can be either a positive or a negative rational number
(d) does not exist

Things to Remember

1. The numbers of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called rational numbers.
2. (i) A rational number is said to be positive if its numerator and denominator are either both positive or both negative.
   (ii) A rational number is said to be negative if its numerator and denominator are of opposite signs.
3. (i) If $\frac{a}{b}$ is a rational number and m is a nonzero integer then $\frac{a}{b} = \frac{a \times m}{b \times m}$.
   (ii) If $\frac{a}{b}$ is a rational number and m is a common divisor of both a and b then $\frac{a}{b} = \frac{a \div m}{b \div m}$.
4. A rational number $\frac{a}{b}$ is said to be in standard form if a and b are integers having no common divisor other than 1 and b is positive.
5. $\frac{a}{b} = \frac{c}{d}$ only when $(a \times d) = (b \times c)$.
6. To compare two or more rational numbers, express each of them as rational number with positive denominator. Take the LCM of these positive denominators and express each rational number with this LCM as denominator. Then, the number having the greater numerator is greater.
7. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then

   (i) $\left(\frac{a}{b} + \frac{c}{d}\right)$ is also a rational number. [closure property]

   (ii) $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$. [commutative law of addition]

   (iii) $\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$. [associative law of addition]

   (iv) $\frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$.

   0 is called the identity element for addition of rational numbers.

   (v) $\left(\frac{a}{b} + \frac{-a}{b}\right) = \left(\frac{-a}{b} + \frac{a}{b}\right) = 0$.

   $-\frac{a}{b}$ is called the additive inverse of $\frac{a}{b}$.
8. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.
9. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then

   (i) $\left(\frac{a}{b} \times \frac{c}{d}\right)$ is also a rational number. [closure property]

   (ii) $\left(\frac{a}{b} \times \frac{c}{d}\right) = \left(\frac{c}{d} \times \frac{a}{b}\right)$. [commutative law of multiplication]

   (iii) $\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$. [associative law of multiplication]

   (iv) $\left(\frac{a}{b} \times 1\right) = \left(1 \times \frac{a}{b}\right) = \frac{a}{b}$.

   1 is called the multiplicative identity for rationals.

   (v) $\left(\frac{a}{b} \times \frac{b}{a}\right) = \left(\frac{b}{a} \times \frac{a}{b}\right) = 1$.

   $\frac{b}{a}$ is called the multiplicative inverse or reciprocal of $\frac{a}{b}$.

(vi) $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} \times \frac{e}{f}\right)$. [*distributive law*]

(vii) $\left(\frac{a}{b} \times 0\right) = \left(0 \times \frac{a}{b}\right) = 0$.

10. *If* $\frac{a}{b}$ *and* $\frac{c}{d}$ *are two rational numbers such that* $\frac{c}{d} \neq 0$ *then* $\left(\frac{a}{b} \div \frac{c}{d}\right) = \left(\frac{a}{b} \times \frac{d}{c}\right)$.

11. (i) *If* $\frac{a}{b}$ *and* $\frac{c}{d}$ *are two rational numbers and* $\frac{c}{d} \neq 0$ *then* $\left(\frac{a}{b} \div \frac{c}{d}\right)$ *is also a rational number.*

(ii) *For every rational number* $\frac{a}{b}$, *we have* $\left(\frac{a}{b} \div 1\right) = \frac{a}{b}$ *and* $\left(\frac{a}{b} \div \frac{a}{b}\right) = 1$.

12. *If* x *and* y *be two rational numbers such that* $x < y$ *then* $\frac{1}{2}(x + y)$ *is a rational number betwee[n]* x *and* y.

TEST PAPER-1

A. **1.** Find the additive inverse of: (i) $\frac{7}{-10}$ (ii) $\frac{8}{5}$.

2. The sum of two rational numbers is –4. If one of them is $\frac{-11}{5}$, find the other.

3. What number should be added to $\frac{-3}{5}$ to get $\frac{2}{3}$?

4. What number should be subtracted from $\frac{-3}{4}$ to get $\frac{-1}{2}$?

5. Find the multiplicative inverse of: (i) $\frac{-3}{4}$ (ii) $\frac{11}{4}$.

6. The product of two numbers is –8. If one of them is –12, find the other.

7. Evaluate:

(i) $\frac{-3}{5} \times \frac{10}{7}$ (ii) $\left(\frac{-5}{8}\right)^{-1}$ (iii) $(-6)^{-1}$

8. Name the property of multiplication shown by each of the following statements:

(i) $\frac{-12}{5} \times \frac{3}{4} = \frac{3}{4} \times \frac{-12}{5}$ (ii) $\frac{-8}{15} \times 1 = \frac{-8}{15}$

(iii) $\left(\frac{-2}{3} \times \frac{7}{8}\right) \times \frac{-5}{7} = \frac{-2}{3} \times \left(\frac{7}{8} \times \frac{-5}{7}\right)$ (iv) $\frac{-2}{3} \times 0 = 0$

(v) $\frac{2}{5} \times \left(\frac{-4}{5} + \frac{-3}{10}\right) = \left(\frac{2}{5} \times \frac{-4}{5}\right) + \left(\frac{2}{5} \times \frac{-3}{10}\right)$

9. Find two rational numbers lying between $\frac{-1}{3}$ and $\frac{1}{2}$.

B. ***Mark (✓) against the correct answer in each of the following:***

10. What should be added to $\frac{-3}{5}$ to get $\frac{-1}{3}$?

(a) $\frac{4}{5}$ (b) $\frac{8}{15}$ (c) $\frac{4}{15}$ (d) $\frac{2}{5}$

11. What should be subtracted from $\frac{-2}{3}$ to get $\frac{3}{4}$?

(a) $\frac{-11}{12}$ (b) $\frac{-13}{12}$ (c) $\frac{-5}{4}$ (d) $\frac{-17}{12}$

12. $\left(\frac{-5}{4}\right)^{-1} = ?$

(a) $\frac{4}{5}$ (b) $\frac{-4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{3}{5}$

13. The product of two numbers is $\frac{-1}{4}$. If one of them is $\frac{-3}{10}$, then the other is

(a) $\frac{5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{4}{3}$ (d) $\frac{-8}{5}$

14. $\left(\frac{-5}{6} \div \frac{-2}{3}\right) = ?$

(a) $\frac{-5}{4}$ (b) $\frac{5}{4}$ (c) $\frac{-4}{5}$ (d) $\frac{4}{5}$

15. $\frac{4}{3} \div ? = \frac{-5}{2}$

(a) $\frac{-8}{5}$ (b) $\frac{8}{5}$ (c) $\frac{-8}{15}$ (d) $\frac{8}{15}$

16. Reciprocal of $\frac{-7}{9}$ is

(a) $\frac{9}{7}$ (b) $\frac{-9}{7}$ (c) $\frac{7}{9}$ (d) none of these

17. A rational number between $\frac{-2}{3}$ and $\frac{1}{2}$ is

(a) $\frac{-1}{6}$ (b) $\frac{-1}{12}$ (c) $\frac{-5}{6}$ (d) $\frac{5}{6}$

C. 18. ***Fill in the blanks.***

(i) $\frac{25}{8} \div (\ldots\ldots) = -10.$

(ii) $\frac{-8}{9} \times (\ldots\ldots) = \frac{-2}{3}.$

(iii) $(-1) + (\ldots\ldots) = \frac{-2}{9}.$

(iv) $\frac{2}{3} - (\ldots\ldots) = \frac{1}{15}.$

D. 19. ***Write 'T' for true and 'F' for false for each of the following:***

(i) Rational numbers are always closed under subtraction.

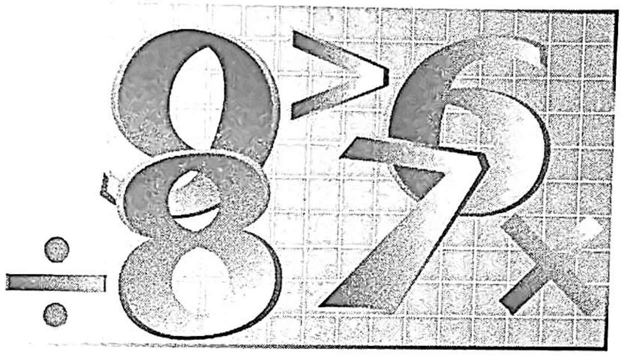
(ii) Rational numbers are always closed under division.

(iii) $1 \div 0 = 0.$

(iv) Subtraction is commutative on rational numbers.

(v) $-\left(\frac{-7}{8}\right) = \frac{7}{8}.$

2 Exponents



Let us recall that for positive integers a and n, we have:

$$(-a)^n = \begin{cases} a^n, & \text{when } n \text{ is even} \\ -a^n, & \text{when } n \text{ is odd.} \end{cases}$$

EXAMPLES (i) $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16 = 2^4.$

(ii) $(-2)^3 = (-2) \times (-2) \times (-2) = -8 = -2^3.$

In this chapter, we shall be dealing with the exponents of rational numbers.

Positive Integral Exponent of a Rational Number

Let $\frac{a}{b}$ be any rational number and n be a positive integer. Then,

$$\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \ldots n \text{ times} = \frac{a \times a \times a \times \ldots n \text{ times}}{b \times b \times b \times \ldots n \text{ times}} = \frac{a^n}{b^n}.$$

Thus, $\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{\boldsymbol{n}} = \frac{\boldsymbol{a}^{\boldsymbol{n}}}{\boldsymbol{b}^{\boldsymbol{n}}}$ for every positive integer n.

EXAMPLE 1. *Evaluate:* *(i)* $\left(\frac{3}{5}\right)^3$ *(ii)* $\left(\frac{-3}{4}\right)^4$ *(iii)* $\left(\frac{-2}{3}\right)^5$

Solution We have:

(i) $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{27}{125}.$

(ii) $\left(\frac{-3}{4}\right)^4 = \frac{(-3)^4}{4^4} = \frac{3^4}{4^4} = \frac{81}{256}.$

(iii) $\left(\frac{-2}{3}\right)^5 = \frac{(-2)^5}{3^5} = \frac{-2^5}{3^5} = \frac{-32}{243}.$

Negative Integral Exponent of a Rational Number

Let $\frac{a}{b}$ be any rational number and n be a positive integer.

Then, we define, $\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{-\boldsymbol{n}} = \left(\frac{\boldsymbol{b}}{\boldsymbol{a}}\right)^{\boldsymbol{n}}.$

EXAMPLES (i) $\left(\frac{3}{4}\right)^{-5} = \left(\frac{4}{3}\right)^5$ (ii) $4^{-6} = \left(\frac{4}{1}\right)^{-6} = \left(\frac{1}{4}\right)^6.$

Also, we define, $\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^0 = \mathbf{1}.$

EXAMPLE 2. *Evaluate:*

(i) $\left(\frac{2}{3}\right)^{-3}$ (ii) 4^{-2} (iii) $\left(\frac{1}{6}\right)^{-2}$ (iv) $\left(\frac{2}{3}\right)^{0}$

Solution We have:

(i) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{3^3}{2^3} = \frac{27}{8}$.

(ii) $4^{-2} = \left(\frac{4}{1}\right)^{-2} = \left(\frac{1}{4}\right)^{2} = \frac{1^2}{4^2} = \frac{1}{16}$.

(iii) $\left(\frac{1}{6}\right)^{-2} = \left(\frac{6}{1}\right)^{2} = 6^2 = 36$.

(iv) $\left(\frac{2}{3}\right)^{0} = 1$.

LAWS OF EXPONENTS

Let $\frac{a}{b}$ be any rational number, and m and n be any integers. Then, we have:

(i) $\left(\frac{a}{b}\right)^{m} \times \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m+n}$.

(ii) $\left(\frac{a}{b}\right)^{m} \div \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m-n}$.

(iii) $\left\{\left(\frac{a}{b}\right)^{m}\right\}^{n} = \left(\frac{a}{b}\right)^{mn}$.

(iv) $\left(\frac{a}{b} \times \frac{c}{d}\right)^{n} = \left(\frac{a}{b}\right)^{n} \times \left(\frac{c}{d}\right)^{n}$ and $\left\{\frac{(a/b)}{(c/d)}\right\}^{n} = \frac{(a/b)^n}{(c/d)^n}$.

(v) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$, where n is a positive integer.

(vi) $\left(\frac{a}{b}\right)^{0} = 1$.

SOLVED EXAMPLES

EXAMPLE 1. *Evaluate:*

(i) 5^{-3} (ii) $\left(\frac{1}{3}\right)^{-4}$ (iii) $\left(\frac{5}{2}\right)^{-3}$ (iv) $(-2)^{-5}$ (v) $\left(\frac{-3}{4}\right)^{-4}$

Solution We have:

(i) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$.

(ii) $\left(\frac{1}{3}\right)^{-4} = \left(\frac{3}{1}\right)^{4} = 3^4 = 81$.

(iii) $\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^{3} = \frac{2^3}{5^3} = \frac{8}{125}$.

(iv) $(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{-2^5} = \frac{1}{-32} = \frac{-1}{32}$.

(v) $\left(\frac{-3}{4}\right)^{-4} = \left(\frac{4}{-3}\right)^4 = \left(\frac{-4}{3}\right)^4 = \frac{(-4)^4}{3^4} = \frac{4^4}{3^4} = \frac{256}{81}$.

EXAMPLE 2. *Evaluate:*

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$ *(ii)* $\left(\frac{4}{7}\right)^5 \times \left(\frac{4}{7}\right)^{-3}$ *(iii)* $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{-2}$ *(iv)* $\left(\frac{8}{5}\right)^{-3} \times \left(\frac{8}{5}\right)^2$

Solution We have:

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^{3+2}$

$= \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$.

(ii) $\left(\frac{4}{7}\right)^5 \times \left(\frac{4}{7}\right)^{-3} = \left(\frac{4}{7}\right)^{5+(-3)}$

$= \left(\frac{4}{7}\right)^2 = \frac{4^2}{7^2} = \frac{16}{49}$.

(iii) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{-2} = \left(\frac{3}{2}\right)^{(-3)+(-2)}$

$= \left(\frac{3}{2}\right)^{-5} = \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$.

(iv) $\left(\frac{8}{5}\right)^{-3} \times \left(\frac{8}{5}\right)^2 = \left(\frac{8}{5}\right)^{(-3)+2}$

$= \left(\frac{8}{5}\right)^{-1} = \left(\frac{5}{8}\right)$.

EXAMPLE 3. *Evaluate* $\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$.

Solution We have:

$\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3} = \left(\frac{8}{3}\right)^2 \times \left(\frac{5}{4}\right)^3$

$= \left(\frac{8^2}{3^2} \times \frac{5^3}{4^3}\right) = \left(\frac{64}{9} \times \frac{125}{64}\right) = \frac{125}{9}$.

EXAMPLE 4. *Evaluate* $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^2$.

Solution We have:

$\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^2 = \left(\frac{7}{-2}\right)^4 \times \left(\frac{-5}{7}\right)^2$

$= \left(\frac{-7}{2}\right)^4 \times \left(\frac{-5}{7}\right)^2$ $\left[\because \left(\frac{7}{-2}\right) = \left(\frac{-7}{2}\right)\right]$

$$= \frac{(-7)^4}{2^4} \times \frac{(-5)^2}{7^2} = \frac{7^4 \times 5^2}{2^4 \times 7^2} \qquad [\because (-7)^4 = 7^4 \text{ and } (-5)^2 = 5^2]$$

$$= \frac{7^2 \times 5^2}{2^4} = \frac{49 \times 25}{16} = \frac{1225}{16}.$$

EXAMPLE 5. *Evaluate* $\left(\frac{-1}{4}\right)^{-3} \times \left(\frac{-1}{4}\right)^{-2}$.

Solution We have:

$$\left(\frac{-1}{4}\right)^{-3} \times \left(\frac{-1}{4}\right)^{-2} = \left(\frac{4}{-1}\right)^{3} \times \left(\frac{4}{-1}\right)^{2} = (-4)^3 \times (-4)^2$$

$$= (-4)^{(3+2)} = (-4)^5 = -4^5 = -1024.$$

EXAMPLE 6. *Evaluate* $\left\{\left(\frac{-3}{2}\right)^{2}\right\}^{-3}$.

Solution We have:

$$\left\{\left(\frac{-3}{2}\right)^{2}\right\}^{-3} = \left(\frac{-3}{2}\right)^{2\times(-3)} = \left(\frac{-3}{2}\right)^{-6} = \left(\frac{2}{-3}\right)^{6}$$

$$= \left(\frac{-2}{3}\right)^{6} = \frac{(-2)^6}{3^6} = \frac{2^6}{3^6} = \frac{64}{729}.$$

EXAMPLE 7. *Simplify:*

(i) $(2^{-1} \times 5^{-1})^{-1} \div 4^{-1}$ (ii) $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Solution We have:

(i) $(2^{-1} \times 5^{-1})^{-1} \div 4^{-1} = \left(\frac{1}{2} \times \frac{1}{5}\right)^{-1} \div \left(\frac{4}{1}\right)^{-1}$

$$= \left(\frac{1}{10}\right)^{-1} \div \frac{1}{4} = \left(\frac{10}{1}\right) \div \left(\frac{1}{4}\right)$$

$$= \left(10 \div \frac{1}{4}\right) = (10 \times 4) = 40.$$

(ii) $(4^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1} = \left(\frac{1}{4} + \frac{1}{8}\right) \div \left(\frac{3}{2}\right)$

$$= \frac{(2+1)}{8} \div \frac{3}{2} = \left(\frac{3}{8} \div \frac{3}{2}\right) = \left(\frac{3}{8} \times \frac{2}{3}\right) = \frac{1}{4}.$$

EXAMPLE 8. *Simplify* $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$.

Solution We have:

$$\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = \left(\frac{2}{1}\right)^{2} + \left(\frac{3}{1}\right)^{2} + \left(\frac{4}{1}\right)^{2}$$

$$= (2^2 + 3^2 + 4^2) = (4 + 9 + 16) = 29.$$

EXAMPLE 9. *By what number should* $\left(\frac{1}{2}\right)^{-1}$ *be multiplied so that the product is* $\left(\frac{-5}{4}\right)^{-1}$?

Solution Let the required number be x. Then,

$$x \times \left(\frac{1}{2}\right)^{-1} = \left(\frac{-5}{4}\right)^{-1}$$

$$\Rightarrow \quad x \times \frac{2}{1} = \frac{4}{-5} \quad \Rightarrow \quad 2x = \frac{-4}{5}$$

$$\Rightarrow \quad x = \left(\frac{1}{2} \times \frac{-4}{5}\right) = \frac{-2}{5}.$$

Hence, the required number is $\frac{-2}{5}$.

EXAMPLE 10. *By what number should* $\left(\frac{-3}{2}\right)^{-3}$ *be divided so that the quotient is* $\left(\frac{9}{4}\right)^{-2}$ *?*

Solution Let the required number be x. Then

$$\frac{\left(\frac{-3}{2}\right)^{-3}}{x} = \left(\frac{9}{4}\right)^{-2} \quad \Rightarrow \quad \left(\frac{-2}{3}\right)^{3} = \left(\frac{4}{9}\right)^{2} \times x$$

$$\Rightarrow \quad \frac{(-2)^3}{3^3} = \frac{4^2}{9^2} \times x \quad \Rightarrow \quad \frac{-8}{27} = \frac{16}{81} \times x$$

$$\Rightarrow \quad x = \left(\frac{-8}{27} \times \frac{81}{16}\right) \quad \Rightarrow \quad x = \frac{-3}{2}.$$

Hence, the required number is $\frac{-3}{2}$.

EXERCISE 2A

1. Evaluate:

(i) 4^{-3} (ii) $\left(\frac{1}{2}\right)^{-5}$ (iii) $\left(\frac{4}{3}\right)^{-3}$ (iv) $(-3)^{-4}$ (v) $\left(\frac{-2}{3}\right)^{-5}$

2. Evaluate:

(i) $\left(\frac{5}{3}\right)^{2} \times \left(\frac{5}{3}\right)^{2}$ (ii) $\left(\frac{5}{6}\right)^{6} \times \left(\frac{5}{6}\right)^{-4}$ (iii) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2}$ (iv) $\left(\frac{9}{8}\right)^{-3} \times \left(\frac{9}{8}\right)^{2}$

3. Evaluate:

(i) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{3}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^{0}$ (ii) $\left(\frac{-3}{5}\right)^{-4} \times \left(\frac{-2}{5}\right)^{2}$ (iii) $\left(\frac{-2}{3}\right)^{-3} \times \left(\frac{-2}{3}\right)^{-2}$

4. Evaluate:

(i) $\left\{\left(\frac{-2}{3}\right)^{2}\right\}^{-2}$ (ii) $\left[\left\{\left(\frac{-1}{3}\right)^{2}\right\}^{-2}\right]^{-1}$ (iii) $\left\{\left(\frac{3}{2}\right)^{-2}\right\}^{2}$

5. Evaluate $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$.

6. Evaluate $\left\{\left(\frac{4}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$.

7. Evaluate $[(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}]$.

8. Find the value of:

(i) $(2^0 + 3^{-1}) \times 3^2$ (ii) $(2^{-1} \times 3^{-1}) \div 2^{-3}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

9. Find the value of x for which $\left(\frac{5}{3}\right)^{-4} \times \left(\frac{5}{3}\right)^{-5} = \left(\frac{5}{3}\right)^{3x}$.

10. Find the value of x for which $\left(\frac{4}{9}\right)^{4} \times \left(\frac{4}{9}\right)^{-7} = \left(\frac{4}{9}\right)^{2x-1}$.

11. By what number should $(-6)^{-1}$ be multiplied so that the product becomes 9^{-1}?

12. By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

13. If $5^{2x+1} \div 25 = 125$, find the value of x.

NUMBERS IN STANDARD FORM *A number written as* $(m \times 10^n)$ *is said to be in standard form if* m *is a decimal number such that* $1 \le m < 10$ *and* n *is either a positive or a negative integer.*

I. EXPRESSING VERY LARGE NUMBERS IN STANDARD FORM

SOLVED EXAMPLES

EXAMPLE 1. *Express each of the following numbers in standard form:*
(i) 6872 *(ii)* 140000 *(iii)* 15360000000

Solution We may write:

(i) $6872 = 6.872 \times 1000 = (6.872 \times 10^3)$.

(ii) $140000 = 14 \times 10000 = (1.4 \times 10 \times 10^4) = (1.4 \times 10^5)$.

(iii) $15360000000 = 1536 \times 10000000 = (1.536 \times 1000 \times 10^7)$
$= (1.536 \times 10^3 \times 10^7) = (1.536 \times 10^{10})$.

EXAMPLE 2. *The diameter of the sun is* (1.4×10^9) m *and the diameter of the earth is* (1.2756×10^7) m. *Show that the diameter of the sun is nearly* 100 *times the diameter of the earth.*

Solution We have:

$$\frac{\text{diameter of the sun}}{\text{diameter of the earth}} = \frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^2}{1.2756} = \frac{14000}{12756} \times 100 = 100 \text{ (nearly)}.$$

$\therefore$ (diameter of the sun) $= 100 \times$ (diameter of the earth).

EXAMPLE 3. *In a stack there are* 4 *books each of thickness* 24 mm *and* 6 *paper sheets each of thickness* 0.015 mm. *What is the total thickness of the stack in standard form?*

Solution Total thickness of the stack

$= (24 \times 4)$ mm $+ (0.015 \times 6)$ mm

$= 96$ mm $+ 0.090$ mm $= (96.090)$ mm

$= (96.09)$ mm $= \frac{9609}{100}$ mm $= \left(\frac{9.609 \times 1000}{100}\right)$ mm

$= \left(\frac{9.609 \times 10^3}{10^2}\right)$ mm $= (9.609 \times 10)$ mm.

Hence, the total thickness of the stack is (9.609×10) mm.

EXAMPLE 4. *The distance between sun and earth is (1.496×10^{11}) m and the distance between earth and moon is (3.84×10^{8}) m. During solar eclipse moon comes in between earth and sun. At that time what is the distance between moon and sun?*

Solution Required distance

$$= \{(1.496 \times 10^{11}) - (3.84 \times 10^{8})\} \text{ m}$$
$$= \left\{\left(\frac{1496 \times 10^{11}}{10^{3}}\right) - (3.84 \times 10^{8})\right\} \text{ m}$$
$$= \{(1496 \times 10^{8}) - (3.84 \times 10^{8})\} \text{ m}$$
$$= \{(1496 - 3.84) \times 10^{8}\} \text{ m} = (1492.16 \times 10^{8}) \text{ m}.$$

Hence, the distance between moon and sun is (1492.16×10^{8}) m.

EXAMPLE 5. *Write each of the following numbers in usual form:*
(i) 4.61×10^{5} *(ii)* 2.514×10^{7} *(iii)* 2.0001×10^{8}

Solution We have:

(i) $4.61 \times 10^{5} = \frac{461}{100} \times 10^{5} = \frac{461 \times 10^{5}}{10^{2}} = 461 \times 10^{(5-2)}$
$= (461 \times 10^{3}) = (461 \times 1000) = 461000.$

(ii) $2.514 \times 10^{7} = \frac{2514}{1000} \times 10^{7} = \frac{2514 \times 10^{7}}{10^{3}} = 2514 \times 10^{(7-3)}$
$= (2514 \times 10^{4}) = (2514 \times 10000) = 25140000.$

(iii) $2.0001 \times 10^{8} = \frac{20001}{10000} \times 10^{8} = \frac{20001 \times 10^{8}}{10^{4}} = 20001 \times 10^{(8-4)}$
$= (20001 \times 10^{4}) = 200010000.$

II. EXPRESSING VERY SMALL NUMBERS IN STANDARD FORM

EXAMPLE 6. *Express each of the following numbers in standard form:*
(i) 0.00002 *(ii)* 0.000000061 *(iii)* 0.00000000837

Solution We may write:

(i) $0.00002 = \frac{2}{10^{5}} = (2 \times 10^{-5}).$

(ii) $0.000000061 = \frac{61}{10^{9}} = \frac{6.1 \times 10}{10^{9}} = \frac{6.1}{10^{8}} = (6.1 \times 10^{-8}).$

(iii) $0.00000000837 = \frac{837}{10^{11}} = \frac{8.37 \times 100}{10^{11}} = \frac{8.37 \times 10^{2}}{10^{11}}$
$= \frac{8.37}{10^{(11-2)}} = \frac{8.37}{10^{9}} = (8.37 \times 10^{-9}).$

EXAMPLE 7. *The size of a red blood cell is 0.000007 m and that of a plant cell is 0.00001275 m. Show that a red blood cell is half of plant cell in size.*

Solution We have:

Size of a red blood cell = 0.000007 m $= \frac{7}{10^{6}}$ m $= (7 \times 10^{-6})$ m.

Size of a plant cell = 0.00001275 m $= \frac{1275}{10^{8}}$ m $= \left(\frac{1.275 \times 10^{3}}{10^{8}}\right)$ m

$= \frac{1.275}{10^{(8-3)}}$ m $= \frac{1.275}{10^{5}}$ m $= (1.275 \times 10^{-5})$ m.

$$\frac{\text{Size of a red blood cell}}{\text{Size of a plant cell}} = \frac{7\times10^{-6}}{1.275\times10^{-5}} = \frac{7\times10^{(-6+5)}}{1.275} = \frac{7\times10^{-1}}{1.275}$$

$$= \frac{7}{(1.275\times10)} = \frac{7}{12.75} = \frac{7}{13} \text{ (nearly)}$$

$$= \frac{1}{2} \text{ (approximately).}$$

$\therefore$ size of a red blood cell $= \frac{1}{2}\times$ (size of a plant cell).

EXAMPLE 8. *Express the following numbers in usual form:*

(i) 2×10^{-5} *(ii)* 6.32×10^{-4} *(iii)* 1.596×10^{-6}

Solution We have:

(i) $2\times10^{-5} = \frac{2}{10^5} = \frac{2}{100000} = 0.00002.$

(ii) $6.32\times10^{-4} = \frac{632}{100}\times\frac{1}{10^4} = \frac{632}{10^2\times10^4} = \frac{632}{10^6}$

$= \frac{632}{1000000} = 0.000632.$

(iii) $1.596\times10^{-6} = \frac{1596\times10^{-6}}{1000} = \frac{1596}{10^3\times10^6} = \frac{1596}{10^9} = 0.000001596.$

EXERCISE 2B

1. Write each of the following numbers in standard form:
   (i) 57.36 (ii) 3500000 (iii) 273000
   (iv) 168000000 (v) 4630000000000 (vi) 345×10^5
2. Write each of the following numbers in usual form:
   (i) 3.74×10^5 (ii) 6.912×10^8 (iii) 4.1253×10^7
   (iv) 2.5×10^4 (v) 5.17×10^6 (vi) 1.679×10^9
3. (i) The height of Mount Everest is 8848 m. Write it in standard form.
   (ii) The speed of light is 300000000 m/sec. Express it in standard form.
   (iii) The distance from the earth to the sun is 149600000000 m. Write it in standard form.
4. Mass of earth is (5.97×10^{24}) kg and mass of moon is (7.35×10^{22}) kg. What is the total mass of the two?

   Hint. *Total mass* $= [(5.97\times10^2\times10^{22}+(7.35\times10^{22})]$ kg
   $= [(597\times10^{22})+(7.35\times10^{22})]$ kg $= [(597+7.35)\times10^{22}]$ kg.
5. Write each of the following numbers in standard form:
   (i) 0.0006 (ii) 0.00000083 (iii) 0.0000000534
   (iv) 0.0027 (v) 0.00000165 (vi) 0.00000000689
6. (i) 1 micron $= \frac{1}{1000000}$ m. Express it in standard form.
   (ii) Size of a bacteria = 0.0000004 m. Express it in standard form.
   (iii) Thickness of a paper = 0.03 mm. Express it in standard form.
7. Write each of the following numbers in usual form:
   (i) 2.06×10^{-5} (ii) 5×10^{-7} (iii) 6.82×10^{-6}
   (iv) 5.673×10^{-4} (v) 1.8×10^{-2} (vi) 4.129×10^{-3}

EXERCISE 2C

OBJECTIVE QUESTIONS

Tick (✓) the correct answer in each of the following:

1. The value of $\left(\frac{2}{5}\right)^{-3}$ is

(a) $-\frac{8}{125}$ (b) $\frac{25}{4}$ (c) $\frac{125}{8}$ (d) $-\frac{2}{5}$

2. The value of $(-3)^{-4}$ is

(a) 12 (b) 81 (c) $-\frac{1}{12}$ (d) $\frac{1}{81}$

3. The value of $(-2)^{-5}$ is

(a) -32 (b) $\frac{-1}{32}$ (c) 32 (d) $\frac{1}{32}$

4. $(2^{-5} \div 2^{-2}) = ?$

(a) $\frac{1}{128}$ (b) $\frac{-1}{128}$ (c) $-\frac{1}{8}$ (d) $\frac{1}{8}$

5. The value of $(3^{-1}+4^{-1})^{-1} \div 5^{-1}$ is

(a) $\frac{7}{10}$ (b) $\frac{60}{7}$ (c) $\frac{7}{5}$ (d) $\frac{7}{15}$

6. $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2} = ?$

(a) $\frac{61}{144}$ (b) $\frac{144}{61}$ (c) 29 (d) $\frac{1}{29}$

7. $\left\{\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3} = ?$

(a) $\frac{19}{64}$ (b) $\frac{27}{16}$ (c) $\frac{64}{19}$ (d) $\frac{16}{25}$

8. $\left[\left\{\left(-\frac{1}{2}\right)^{2}\right\}^{-2}\right]^{-1} = ?$

(a) $\frac{1}{16}$ (b) 16 (c) $-\frac{1}{16}$ (d) -16

9. The value of x for which $\left(\frac{7}{12}\right)^{-4} \times \left(\frac{7}{12}\right)^{3x} = \left(\frac{7}{12}\right)^{5}$, is

(a) -1 (b) 1 (c) 2 (d) 3

10. If $(2^{3x-1}+10) \div 7 = 6$, then x is equal to

(a) -2 (b) 0 (c) 1 (d) 2

11. $\left(\frac{2}{3}\right)^{0} = ?$

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) 0

12. $\left(\frac{-5}{3}\right)^{-1} = ?$

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{-3}{5}$ (d) none of these

13. $\left(-\frac{1}{2}\right)^{3} = ?$

(a) $\frac{-1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{8}$ (d) $\frac{-1}{8}$

14. $\left(\frac{-3}{4}\right)^{2} = ?$

(a) $\frac{-9}{16}$ (b) $\frac{9}{16}$ (c) $\frac{16}{9}$ (d) $\frac{-16}{9}$

15. 3670000 in standard form is

(a) 367×10^4 (b) 36.7×10^5 (c) 3.67×10^6 (d) none of these

16. 0.0000463 in standard form is

(a) 463×10^{-7} (b) 4.63×10^{-5} (c) 4.63×10^{-9} (d) 46.3×10^{-6}

17. 0.000367×10^4 in usual form is

(a) 3.67 (b) 36.7 (c) 0.367 (d) 0.0367

Things to Remember

I. *Let* $\frac{a}{b}$ *be any rational number, and m and n be any integers. Then, we have:*

1. $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$.
2. $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n}$.
3. $\left\{\left(\frac{a}{b}\right)^m\right\}^n = \left(\frac{a}{b}\right)^{mn}$.
4. $\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$.
5. $\left\{\frac{(a/b)}{(c/d)}\right\}^n = \frac{(a/b)^n}{(c/d)^n}$.
6. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.
7. $\left(\frac{a}{b}\right)^0 = 1$.

II. ***Numbers in standard form:*** *A number written as* $(m \times 10^n)$ *is said to be in standard form if m is a decimal number such that* $1 \le m < 10$ *and n is either a positive or a negative integer.*

EXAMPLES (i) $160000 = (1.6 \times 10^5)$

(ii) $1548000 = (1.548 \times 10^6)$

TEST PAPER-2

A. 1. Evaluate:

(i) 3^{-4} (ii) $(-4)^3$ (iii) $\left(\frac{3}{4}\right)^{-2}$ (iv) $\left(\frac{-2}{3}\right)^{-5}$ (v) $\left(\frac{5}{7}\right)^0$

2. Evaluate: $\left\{\left(\frac{-2}{3}\right)^3\right\}^{-2}$.

3. Simplify: $(3^{-1}+6^{-1}) \div \left(\frac{3}{4}\right)^{-1}$.

4. By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient is $\left(\frac{4}{9}\right)^{-2}$?

5. By what number should $(-3)^{-1}$ be multiplied so that the product becomes 6^{-1} ?

6. Express each of the following in standard form:

(i) 345 (ii) 180000 (iii) 0.000003 (iv) 0.000027

B. ***Mark (✓) against the correct answer in each of the following:***

7. The value of $(-3)^{-3}$ is

(a) –27 (b) 9 (c) $\frac{-1}{27}$ (d) $\frac{1}{27}$

8. The value of $\left(\frac{3}{4}\right)^{-3}$ is

(a) $\frac{-27}{64}$ (b) $\frac{64}{27}$ (c) $\frac{-9}{4}$ (d) $\frac{27}{64}$

9. $(3^{-6} \div 3^4) = ?$

(a) 3^{-2} (b) 3^2 (c) 3^{-10} (d) 3^{10}

10. If $\left(\frac{5}{12}\right)^{-4} \times \left(\frac{5}{12}\right)^{3x} = \left(\frac{5}{12}\right)^5$, then $x = ?$

(a) –1 (b) 1 (c) 2 (d) 3

11. $\left(\frac{3}{5}\right)^0 = ?$

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) 1 (d) 0

12. $\left(\frac{-6}{5}\right)^{-1} = ?$

(a) $\frac{6}{5}$ (b) $\frac{-6}{5}$ (c) $\frac{5}{6}$ (d) $\frac{-5}{6}$

13. $\left(\frac{-1}{3}\right)^3 = ?$

(a) $\frac{-1}{9}$ (b) $\frac{1}{9}$ (c) $\frac{-1}{27}$ (d) $\frac{1}{27}$

C. 14. *Fill in the blanks.*

(i) 360000 written in standard form is ....

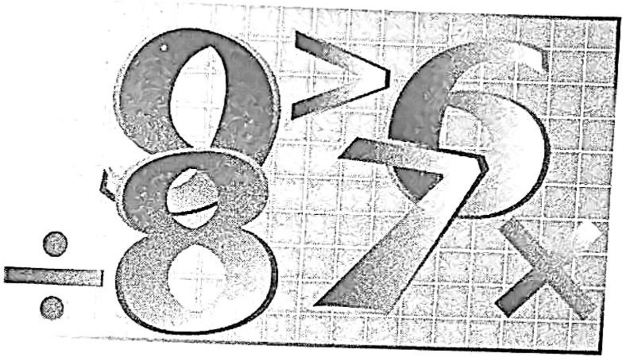
(ii) 0.0000123 written in standard form is ....

(iii) $\left(\frac{-2}{3}\right)^{-2}$ = ...

(iv) 3×10^{-3} in usual form is ....

(v) 5.32×10^{-4} in usual form is ....

3 Squares and Square Roots



SQUARES

Square *The square of a number is the product of the number with the number itself. For a given number x, the square of x is $(x \times x)$, denoted by x^2.*

EXAMPLES (i) $4^2 = (4 \times 4) = 16$ and we say that the square of 4 is 16.

(ii) $9^2 = (9 \times 9) = 81$ and we say that the square of 9 is 81.

PERFECT SQUARES OR SQUARE NUMBERS

A natural number is called a perfect square or a square number if it is the square of some natural number.

EXAMPLES We know that $1 = 1^2$; $4 = 2^2$; $9 = 3^2$; $16 = 4^2$; $25 = 5^2$, and so on.

Thus 1, 4, 9, 16, 25, etc., are perfect squares.

Test: *A perfect square number is always expressible as the product of pairs of equal factors.*

EXAMPLE 1. *Is 196 a perfect square? If so, find the number whose square is 196.*

Solution Resolving 196 into prime factors, we get

$196 = \underline{2 \times 2} \times \underline{7 \times 7}$.

2	196
2	98
7	49
7	7
	1

Thus, 196 can be expressed as a product of pairs of equal factors.

$\therefore$ 196 is a perfect square.

Also, $196 = (2^2 \times 7^2) = (2 \times 7)^2 = (14)^2$.

Hence, 14 is the number whose square is 196.

EXAMPLE 2. *Show that 1764 is a perfect square. Find the number whose square is 1764.*

Solution Resolving 1764 into prime factors, we get

$1764 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$.

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Thus, 1764 is the product of pairs of equal factors.

$\therefore$ 1764 is a perfect square.

Also, $1764 = (2^2 \times 3^2 \times 7^2) = (2 \times 3 \times 7)^2 = (42)^2$.

Hence, 42 is the number whose square is 1764.

EXAMPLE 3. *Show that 6292 is not a perfect square.*

2	6292
2	3146
11	1573
11	143
13	13
	1

Solution Resolving 6292 into prime factors, we get

$$6292 = \underline{2 \times 2} \times \underline{11 \times 11} \times 13 = (2^2 \times 11^2 \times 13).$$

Thus, 6292 cannot be expressed as a product of pairs of equal factors.

Hence, 6292 is not a perfect square.

EXAMPLE 4. *By what least number should 3675 be multiplied to get a perfect square number? Also, find the number whose square is the new number.*

3	3675
5	1225
5	245
7	49
7	7
	1

Solution Resolving 3675 into prime factors, we get

$$3675 = 3 \times \underline{5 \times 5} \times \underline{7 \times 7} = (3 \times 5^2 \times 7^2).$$

Thus, to get a perfect square number, the given number should be multiplied by 3.

New number $= (3^2 \times 5^2 \times 7^2) = (3 \times 5 \times 7)^2 = (105)^2$

Hence, the number whose square is the new number = 105.

EXAMPLE 5. *By what least number should 6300 be divided to get a perfect square number? Find the number whose square is the new number.*

3	6300
3	2100
7	700
5	100
5	20
2	4
2	2
	1

Solution Resolving 6300 into prime factors, we get

$$6300 = \underline{3 \times 3} \times 7 \times \underline{5 \times 5} \times \underline{2 \times 2} = (3^2 \times 7 \times 5^2 \times 2^2).$$

Thus, to get a perfect square number, the given number should be divided by 7.

New number obtained $= (3^2 \times 5^2 \times 2^2) = (3 \times 5 \times 2)^2 = (30)^2$

Hence, the number whose square is the new number = 30.

EXERCISE 3A

1. Using the prime factorisation method, find which of the following numbers are perfect squares:
   (i) 441 (ii) 576 (iii) 11025 (iv) 1176
   (v) 5625 (vi) 9075 (vii) 4225 (viii) 1089
2. Show that each of the following numbers is a perfect square. In each case, find the number whose square is the given number:
   (i) 1225 (ii) 2601 (iii) 5929 (iv) 7056
   (v) 8281
3. By what least number should the given number be multiplied to get a perfect square number? In each case, find the number whose square is the new number.
   (i) 3675 (ii) 2156 (iii) 3332 (iv) 2925
   (v) 9075 (vi) 7623 (vii) 3380 (viii) 2475
4. By what least number should the given number be divided to get a perfect square number? In each case, find the number whose square is the new number.
   (i) 1575 (ii) 9075 (iii) 4851 (iv) 3380
   (v) 4500 (vi) 7776 (vii) 8820 (viii) 4056
5. Find the largest number of 2 digits which is a perfect square.
6. Find the largest number of 3 digits which is a perfect square.

PROPERTIES OF PERFECT SQUARES

Property 1. *A number ending in 2, 3, 7 or 8 is never a perfect square.*

EXAMPLES The numbers 82, 93, 187, 248 end in 2, 3, 7, 8 respectively.
So, none of them is a perfect square.

Property 2. *A number ending in an odd number of zeros is never a perfect square.*

EXAMPLES The numbers 160, 4000, 900000 end in one zero, three zeros and five zeros respectively.
So, none of them is a perfect square.

Property 3. *If a number when divided by 3 leaves a remainder 2, then it is not a perfect square.*

EXAMPLES 170, 578, 617, 722, etc.

Property 4. *If a number when divided by 4 leaves a remainder 2 or 3, then it is not a perfect square.*

EXAMPLES 578, 654, 798, 1002, etc.

Property 5. *The square of an even number is always even.*

EXAMPLES $2^2 = 4, 4^2 = 16, 6^2 = 36, 8^2 = 64$, etc.

Property 6. *The square of an odd number is always odd.*

EXAMPLES $1^2 = 1, 3^2 = 9, 5^2 = 25, 7^2 = 49, 9^2 = 81$, etc.

Property 7. *The square of a proper fraction is smaller than the fraction.*

EXAMPLE $\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3} \times \frac{2}{3}\right) = \frac{4}{9}$ and $\frac{4}{9} < \frac{2}{3}$, since $(4 \times 3) < (9 \times 2)$.

Property 8. For every natural number n, we have

$$(n+1)^2 - n^2 = (n+1+n)(n+1-n) = \{(n+1)+n\}.$$

$$\therefore \quad \{(n+1)^2 - n^2\} = \{(n+1)+n\}.$$

EXAMPLES (i) $\{(36)^2 - (35)^2\} = (36+35) = 71$ (ii) $\{(89)^2 - (88)^2\} = (89+88) = 177$.

Property 9. For every natural number n, we have

sum of first n odd natural numbers $= n^2$.

EXAMPLES (i) $\{1+3+5+7+9\}$ = sum of first 5 odd numbers $= 5^2$.
(ii) $\{1+3+5+7+9+11+13+15\}$ = sum of first 8 odd numbers $= 8^2$.

Property 10 (Pythagorean Triplets): *Three natural numbers m, n, p are said to form a Pythagorean triplet (m, n, p) if* $(m^2+n^2) = p^2$.

An Important Result For every natural number $m > 1$, we have $(2m, m^2-1, m^2+1)$ as a Pythagorean triplet.

EXAMPLES (i) Putting $m = 4$ in $(2m, m^2-1, m^2+1)$, we get (8, 15, 17) as a Pythagorean triplet.
(ii) Putting $m = 5$ in $(2m, m^2-1, m^2+1)$, we get (10, 24, 26) as a Pythagorean triplet.

Property 11. *Between two consecutive square numbers* n^2 *and* $(n+1)^2$*, there are* $2n$ *non-perfect square numbers.*

SOLVED EXAMPLES

EXAMPLE 1. *Give reason to show that none of the numbers given below is a perfect square:*
(i) 2162 *(ii)* 6843 *(iii)* 9637 *(iv)* 6598

Solution We know that a number ending in 2, 3, 7 or 8 is never a perfect square.
Hence, none of the numbers 2162, 6843, 9637 and 6598 is a perfect square.

EXAMPLE 2. *Give reason to show that none of the numbers* 640, 81000 *and* 3600000 *is a perfect square.*

Solution We know that a number ending in an odd number of zeros is never a perfect square.
So, none of the numbers 640, 81000 and 3600000 is a perfect square.

EXAMPLE 3. *State whether the square of the given number is even or odd:*
(i) 523 *(ii)* 654 *(iii)* 6776 *(iv)* 7025

Solution We know that the square of an odd number is odd and the square of an even number is even.
(i) 523 is odd $\Rightarrow$ $(523)^2$ is odd.
(ii) 654 is even $\Rightarrow$ $(654)^2$ is even.
(iii) 6776 is even $\Rightarrow$ $(6776)^2$ is even.
(iv) 7025 is odd $\Rightarrow$ $(7025)^2$ is odd.

EXAMPLE 4. *Without adding, find the sum:*
(i) $(1+3+5+7+9+11)$ *(ii)* $(1+3+5+7+9+11+13+15+17)$

Solution We have
(i) $(1+3+5+7+9+11)$ = sum of first 6 odd numbers $= 6^2 = 36$.
(ii) $(1+3+5+7+9+11+13+15+17)$ = sum of first 9 odd numbers $= 9^2 = 81$.

EXAMPLE 5. *Express:*
(i) 64 *as the sum of* 8 *odd natural numbers.*
(ii) 121 *as the sum of* 11 *odd natural numbers.*

Solution We know that n^2 is equal to the sum of first n odd natural numbers.
(i) $64 = 8^2$ = sum of 8 odd natural numbers $= (1+3+5+7+9+11+13+15)$.
(ii) $121 = (11)^2$ = sum of first 11 odd natural numbers
$= (1+3+5+7+9+11+13+15+17+19+21)$.

EXAMPLE 6. *Find the Pythagorean triplet whose smallest member is* 12.

Solution For every natural number $m > 1$, $(2m, m^2-1, m^2+1)$ is a Pythagorean triplet.
Putting $2m = 12$, we get $m = 6$.
Thus, we get the triplet (12, 35, 37).

EXAMPLE 7. *Evaluate* $[(337)^2 - (336)^2]$.

Solution We have $(n+1)^2 - n^2 = (n+1) + n$.
Taking $n = 336$ and $(n+1) = 337$, we get
$$(337)^2 - (336)^2 = (337 + 336) = 673.$$

EXAMPLE 8. *Using the identity* $(a+b)^2 = (a^2 + 2ab + b^2)$, *evaluate:*
(i) $(609)^2$ *(ii)* $(725)^2$

Solution We have:

(i) $(609)^2 = (600+9)^2 = (600)^2 + 2 \times 600 \times 9 + (9)^2$
$= (360000 + 10800 + 81) = 370881.$

(ii) $(725)^2 = (700+25)^2 = (700)^2 + 2 \times 700 \times 25 + (25)^2$
$= (490000 + 35000 + 625) = 525625.$

EXAMPLE 9. *Using the identity* $(a-b)^2 = (a^2 - 2ab + b^2)$, *evaluate:*
(i) $(491)^2$ *(ii)* $(289)^2$

Solution We have:

(i) $(491)^2 = (500-9)^2 = (500)^2 - 2 \times 500 \times 9 + (9)^2$
$= (250000 - 9000 + 81) = 241081.$

(ii) $(289)^2 = (300-11)^2 = (300)^2 - 2 \times 300 \times 11 + (11)^2$
$= (90000 - 6600 + 121) = 83521.$

PRODUCT OF TWO CONSECUTIVE ODD OR CONSECUTIVE EVEN NUMBERS

EXAMPLE 10. *Evaluate (i)* 49×51 *(ii)* 30×32.

Solution We have:

(i) $49 \times 51 = (50-1) \times (50+1)$
$= [(50)^2 - 1^2] = (2500 - 1) = 2499.$

(ii) $30 \times 32 = (31-1) \times (31+1)$
$= [(31)^2 - 1^2] = (961 - 1) = 960.$

SUMMARY

PROPERTIES OF PERFECT SQUARES

1. A number ending in 2, 3, 7 or 8 is never a perfect square.
2. A number ending in an odd number of zeros is never a perfect square.
3. The square of an even number is even.
4. The square of an odd number is odd.
5. The square of a proper fraction is smaller than the fraction.
6. For every natural number n, we have $\{(n+1)^2 - n^2\} = \{(n+1) + n\}$.
7. Sum of first n odd natural numbers $= n^2$.
8. If m, n, p are natural numbers such that $(m^2 + n^2) = p^2$, then (m, n, p) is called a Pythagorean triplet.
9. For every natural number $m > 1$, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.
10. (i) $(a+b)^2 = (a^2 + b^2 + 2ab)$ (ii) $(a-b)^2 = (a^2 + b^2 - 2ab)$

EXERCISE 3B

1. Give reason to show that none of the numbers given below is a perfect square:
   (i) 5372 (ii) 5963 (iii) 8457 (iv) 9468
   (v) 360 (vi) 64000 (vii) 2500000
2. Which of the following are squares of even numbers?
   (i) 196 (ii) 441 (iii) 900 (iv) 625
   (v) 324

3. Which of the following are squares of odd numbers?

(i) 484 (ii) 961 (iii) 7396 (iv) 8649

(v) 4225

4. Without adding, find the sum:

(i) $(1+3+5+7+9+11+13)$ (ii) $(1+3+5+7+9+11+13+15+17+19)$

(iii) $(1+3+5+7+9+11+13+15+17+19+21+23)$

5. (i) Express 81 as the sum of 9 odd numbers.

(ii) Express 100 as the sum of 10 odd numbers.

6. Write a Pythagorean triplet whose smallest member is

(i) 6 (ii) 14 (iii) 16 (iv) 20

7. Evaluate:

(i) $(38)^2-(37)^2$ (ii) $(75)^2-(74)^2$ (iii) $(92)^2-(91)^2$

(iv) $(105)^2-(104)^2$ (v) $(141)^2-(140)^2$ (vi) $(218)^2-(217)^2$

8. Using the formula $(a+b)^2=(a^2+2ab+b^2)$, evaluate:

(i) $(310)^2$ (ii) $(508)^2$ (iii) $(630)^2$

9. Using the formula $(a-b)^2=(a^2-2ab+b^2)$, evaluate:

(i) $(196)^2$ (ii) $(689)^2$ (iii) $(891)^2$

10. Evaluate: (i) 69×71 (ii) 94×106.

11. Evaluate: (i) 88×92 (ii) 78×82.

12. Fill in the blanks:

(i) The square of an even number is ....

(ii) The square of an odd number is ....

(iii) The square of a proper fraction is ...... than the given fraction.

(iv) n^2 = the sum of first n ...... natural numbers.

13 Write (T) for true and (F) for false for each of the statements given below:

(i) The number of digits in a perfect square is even.

(ii) The square of a prime number is prime.

(iii) The sum of two perfect squares is a perfect square.

(iv) The difference of two perfect squares is a perfect square.

(v) The product of two perfect squares is a perfect square.

SHORT-CUT METHODS FOR SQUARING A NUMBER

1. Column method for squaring a two-digit number

Let the given number have the tens digit = a and the units digit = b.
Then, we have to square this number.

Step 1. Make three columns, I, II and III, headed by a^2, $(2\times a\times b)$ and b^2 respectively. Write the values of a^2, $(2\times a\times b)$ and b^2 in columns I, II and III respectively.

Step 2. In Column III, underline the units digit of b^2 and carry over the tens digit of it to Column II and add it to the value of $(2\times a\times b)$.

Step 3. In Column II, underline the units digit of the number obtained in Step 2 and carry over the tens digit of it to Column I and add it to the value of a^2.

Step 4. Underline the number obtained in Step 3 in Column I. The underlined digits give the required square number.

EXAMPLE 1. *Find the square of* (i) 47 (ii) 86.

Solution (i) Given number = 47.

$\therefore$ $a = 4$ and $b = 7$.

I	II	III
a^2	$(2 \times a \times b)$	b^2
16 +6 = 22	56 +4 = 60	49

$\therefore$ $(47)^2 = 2209$.

(ii) Given number = 86.

$\therefore$ $a = 8$ and $b = 6$.

I	II	III
a^2	$(2 \times a \times b)$	b^2
64 +9 = 73	96 +3 = 99	36

$\therefore$ $(86)^2 = 7396$.

2. Diagonal method for squaring a number

EXAMPLE 2. *Find the square of* 39 *by using the diagonal method.*

Solution

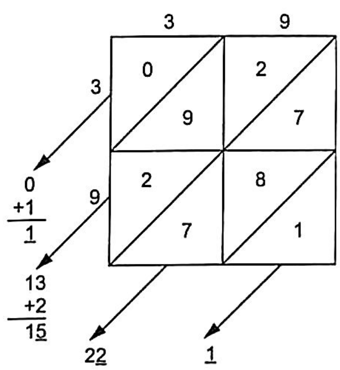
Step 1. The given number contains two digits. So, draw a square and divide it into 4 subsquares as shown below. Write down the digits 3 and 9, horizontally and vertically, as shown below.

Step 2. Multiply each digit on the left of the square with each digit on the top, one by one. Write the product in the corresponding subsquare.
If the product is a one-digit number, write it below the diagonal and put 0 above the diagonal.
In case the product is a two-digit number, write the tens digit above the diagonal and the units digit below the diagonal.

Step 3. Starting below the lowest diagonal, sum the digits diagonally.
If the sum is a two-digit number, underline the units digit and carry over the tens digit to the next diagonal.

Step 4. Underline all the digits in the sum above the topmost diagonal.

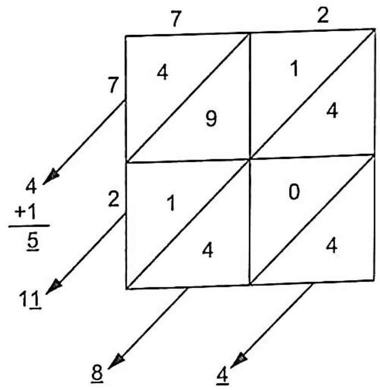
Step 5. The underlined digits give the required square number.



$\therefore$ $(39)^2 = 1521$.

EXAMPLE 3. *Find the square of 72 by using the diagonal method.*

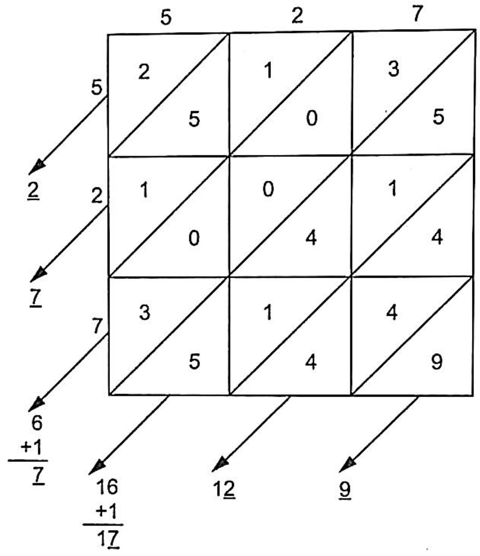
Solution



$\therefore \quad (72)^2 = 5184.$

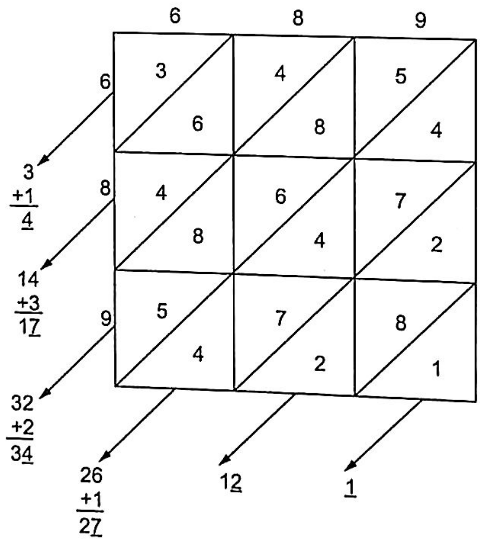
EXAMPLE 4. *Find the square of 527 by using the diagonal method.*

Solution



$\therefore \quad (527)^2 = 277729.$

EXAMPLE 5. *Find the square of 689 by using the diagonal method.*



$\therefore \quad (689)^2 = 474721.$

EXERCISE 3C

Find the value of each of the following, using the column method:

1. $(23)^2$ **2.** $(35)^2$ **3.** $(52)^2$ **4.** $(96)^2$

Find the value of each of the following, using the diagonal method:

5. $(67)^2$ **6.** $(86)^2$ **7.** $(137)^2$ **8.** $(256)^2$

SQUARE ROOTS

Square root *The square root of a number x is that number which when multiplied by itself gives x as the product.*

We denote the square root of a number x by $\sqrt{x}$.

Clearly, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$, $\sqrt{25} = 5$, etc.

Square Root of a Perfect Square by the Prime Factorisation Method

When a given number is a perfect square, we find its square root by the following steps:

Step 1. *Resolve the given number into prime factors.*

Step 2. *Make pairs of similar factors.*

Step 3. *Take the product of the prime factors, choosing one factor out of every pair.*

ILLUSTRATIVE EXAMPLES

EXAMPLE 1. *Find the square root of* 324.

Solution By prime factorisation, we get

$$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}.$$

$$\therefore \quad \sqrt{324} = (2 \times 3 \times 3) = 18.$$

2	324
2	162
3	81
3	27
3	9
3	3
	1

EXAMPLE 2. *Find the square root of* 1764.

Solution By prime factorisation, we get

$$1764 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}.$$

$$\therefore \quad \sqrt{1764} = 2 \times 3 \times 7 = 42.$$

2	1764
2	882
3	441
3	147
7	49
7	7
	1

EXAMPLE 3. *Evaluate* $\sqrt{4356}$.

Solution By prime factorisation, we get

$$4356 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{11 \times 11}.$$

$$\therefore \quad \sqrt{4356} = (2 \times 3 \times 11) = 66.$$

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

EXAMPLE 4. *Evaluate* $\sqrt{11025}$.

Solution By prime factorisation, we get

$$11025 = \underline{5 \times 5} \times \underline{3 \times 3} \times \underline{7 \times 7}.$$

$$\therefore \quad \sqrt{11025} = (5 \times 3 \times 7) = 105.$$

5	11025
5	2205
3	441
3	147
7	49
7	7
	1

EXAMPLE 5. *In an auditorium, the number of rows is equal to the number of chairs in each row. If the capacity of the auditorium is 2025, find the number of chairs in each row.*

Solution Let the number of chairs in each row be x.

Then, the number of rows = x.

Total number of chairs in the auditorium = $(x \times x) = x^2$.

But, the capacity of the auditorium = 2025 (given).

$\therefore \quad x^2 = 2025$

$= \underline{5 \times 5} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\Rightarrow \quad x = (5 \times 3 \times 3) = 45.$

Hence, the number of chairs in each row = 45.

5	2025
5	405
3	81
3	27
3	9
3	3
	1

EXAMPLE 6. *Find the smallest number by which 396 must be multiplied so that the product becomes a perfect square.*

Solution By prime factorisation, we get

$396 = \underline{2 \times 2} \times \underline{3 \times 3} \times 11.$

It is clear that in order to get a perfect square, one more 11 is required.

So, the given number should be multiplied by 11 to make the product a perfect square.

2	396
2	198
3	99
3	33
11	11
	1

EXAMPLE 7. *Find the least square number divisible by each one of 8, 9 and 10.*

Solution The least number divisible by each one of 8, 9, 10 is their LCM.

2	8, 9, 10
	4, 9, 5

Now, LCM of 8, 9, 10 = $(2 \times 4 \times 9 \times 5) = 360$.

By prime factorisation, we get

$360 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 5$

To make it a perfect square it must be multiplied by (2×5), i.e., 10.

Hence, required number = $(360 \times 10) = 3600$.

2	360
2	180
2	90
3	45
3	15
5	5
	1

EXERCISE 3D

Find the square root of each of the following numbers by using the method of prime factorisation:

1. 225 **2.** 441 **3.** 729 **4.** 1296

5. 2025 **6.** 4096 **7.** 7056 **8.** 8100

9. 9216 **10.** 11025 **11.** 15876 **12.** 17424

13. Find the smallest number by which 252 must be multiplied to get a perfect square. Also, find the square root of the perfect square so obtained.

14. Find the smallest number by which 2925 must be divided to obtain a perfect square. Also, find the square root of the perfect square so obtained.

15. 1225 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

16. The students of a class arranged a picnic. Each student contributed as many rupees as the number of students in the class. If the total contribution is ₹ 1156, find the strength of the class.

17. Find the least square number which is exactly divisible by each of the numbers 6, 9, 15 and 20.

18. Find the least square number which is exactly divisible by each of the numbers 8, 12, 15 and 20.

SQUARE ROOT OF A PERFECT SQUARE BY THE LONG-DIVISION METHOD

When numbers are very large, the method of finding their square roots by factorisation becomes lengthy and difficult. So, we use the long-division method which is explained in the following steps.

Long-Division Method for Finding Square Roots

Step 1. *Group the digits in pairs, starting with the digit in the units place. Each pair and the remaining digit (if any) is called a period.*

Step 2. *Think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor and also as the quotient.*

Step 3. *Subtract the product of the divisor and the quotient from the first period and bring down the next period to the right of the remainder. This becomes the new dividend.*

Step 4. *Now, the new divisor is obtained by taking two times the quotient and annexing with it a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and this digit is equal to or just less than the new dividend.*

Step 5. *Repeat steps (2), (3) and (4) till all the periods have been taken up. Now, the quotient so obtained is the required square root of the given number.*

SOLVED EXAMPLES

EXAMPLE 1. *Find the square root of 784 by the long-division method.*

Solution Marking periods and using the long-division method, we have:

2	$\overline{7}\ \overline{84}$(28
	−4
48	384
	−384
	×

$\therefore\ \sqrt{784} = 28.$

EXAMPLE 2. *Evaluate* $\sqrt{5329}$ *using long-division method.*

Solution Marking periods and using the long-division method, we have:

7	$\overline{53}\ \overline{29}$ (73
	−49
143	429
	−429
	×

$\therefore\ \sqrt{5329} = 73.$

EXAMPLE 3. *Evaluate* $\sqrt{16384}$.

Solution Marking periods and using the long-division method, we have:

```
     1 | 1 63 84 (128
       | -1
       |-----------
    22 |   63
       |  -44
       |-----------
   248 |   1984
       |  -1984
       |-----------
       |     ×
```

$\therefore \quad \sqrt{16384} = 128.$

EXAMPLE 4. *Evaluate* $\sqrt{10609}$.

Solution Marking periods and using the long-division method, we have:

```
     1 | 1 06 09 (103
       | -1
       |-----------
   203 |    609
       |   -609
       |-----------
       |     ×
```

$\therefore \quad \sqrt{10609} = 103.$

EXAMPLE 5. *Evaluate* $\sqrt{66049}$.

Solution Marking periods and using the long-division method, we have:

```
     2 | 6 60 49 (257
       | -4
       |-----------
    45 |  260
       | -225
       |-----------
   507 |   3549
       |  -3549
       |-----------
       |     ×
```

$\therefore \quad \sqrt{66049} = 257.$

EXAMPLE 6. *Find the cost of erecting a fence around a square field whose area is 9 hectares if fencing costs ₹ 35 per metre.*

Solution Area of the square field = $(9 \times 10000)\ m^2 = 90000\ m^2$.

Length of each side of the field = $\sqrt{90000}$ m = 300 m.

Perimeter of the field = (4×300) m = 1200 m.

Cost of fencing = ₹ (1200×35) = ₹ 42000.

EXAMPLE 7. *What least number must be subtracted from 7250 to get a perfect square? Also find the square root of this perfect square.*

Solution Let us try to find the square root of 7250.

$$\begin{array}{r|l} 8 & \overline{72}\ \overline{50}\ (85 \\ & -64 \\ \cline{2-2} 165 & \ \ 850 \\ & -825 \\ \cline{2-2} & \ \ \ 25 \end{array}$$

This shows that $(85)^2$ is less than 7250 by 25.

So, the least number to be subtracted from 7250 is 25.

Required perfect square number = $(7250 - 25) = 7225$.

And, $\sqrt{7225} = 85$.

EXAMPLE 8. *Find the greatest number of four digits which is a perfect square.*

Solution Greatest number of four digits = 9999.

Let us try to find the square root of 9999.

$$\begin{array}{r|l} 9 & \overline{99}\ \overline{99}\ (99 \\ & -81 \\ \cline{2-2} 189 & \ \ 1899 \\ & -1701 \\ \cline{2-2} & \ \ \ 198 \end{array}$$

This shows that $(99)^2$ is less than 9999 by 198.

So, the least number to be subtracted is 198.

Hence, the required number is $(9999 - 198) = 9801$.

EXAMPLE 9. *What least number must be added to 5607 to make the sum a perfect square? Find this perfect square and its square root.*

Solution We try to find out the square root of 5607.

$$\begin{array}{r|l} 7 & \overline{56}\ \overline{07}\ (74 \\ & -49 \\ \cline{2-2} 144 & \ \ 707 \\ & -576 \\ \cline{2-2} & \ \ 131 \end{array}$$

We observe here that $(74)^2 < 5607 < (75)^2$.

The required number to be added = $(75)^2 - 5607 = (5625 - 5607) = 18$.

Clearly, the required perfect square = 5625 and $\sqrt{5625} = 75$.

EXAMPLE 10. *Find the least number of six digits which is a perfect square. Find the square root of this number.*

Solution The least number of six digits = 100000, which is not a perfect square.

Now, we must find the least number which when added to 100000 gives a perfect square. This perfect square is the required number.

Now, we find out the square root of 100000.

3	$\overline{10}\ \overline{00}\ \overline{00}$ (316
	−9
61	100
	−61
626	3900
	−3756
	144

Clearly, $(316)^2 < 100000 < (317)^2$.

$\therefore$ the least number to be added $= (317)^2 - 100000 = 489$.

Hence, the required number $= (100000 + 489) = 100489$.

Also, $\sqrt{100489} = 317$.

EXERCISE 3E

Evaluate:

1. $\sqrt{576}$	**2.** $\sqrt{1444}$	**3.** $\sqrt{4489}$	**4.** $\sqrt{6241}$
5. $\sqrt{7056}$	**6.** $\sqrt{9025}$	**7.** $\sqrt{11449}$	**8.** $\sqrt{14161}$
9. $\sqrt{10404}$	**10.** $\sqrt{17956}$	**11.** $\sqrt{19600}$	**12.** $\sqrt{92416}$

13. Find the least number which must be subtracted from 2509 to make it a perfect square.

14. Find the least number which must be subtracted from 7581 to obtain a perfect square. Find this perfect square and its square root.

15. Find the least number which must be added to 6203 to obtain a perfect square. Find this perfect square and its square root.

16. Find the least number which must be added to 8400 to obtain a perfect square. Find this perfect square and its square root.

17. Find the least number of four digits which is a perfect square. Also find the square root of the number so obtained.

18. Find the greatest number of five digits which is a perfect square. Also find the square root of the number so obtained.

19. The area of a square field is 60025 m^2. A man cycles along its boundary at 18 km/h. In how much time will he return to the starting point?

SQUARE ROOTS OF NUMBERS IN DECIMAL FORM

Method: *Make the number of decimal places even by affixing a zero, if necessary. Now, mark periods and find out the square root by the long-division method. Put the decimal point in the square root as soon as the integral part is exhausted.*

SOLVED EXAMPLES

EXAMPLE 1. *Evaluate* $\sqrt{42.25}$.

Solution Using the division method we may find the square root of the given number as shown below.

```
    6 | 42.25 (6.5
      |-36
  125 |  625
      | -625
      |   ×
```

$\therefore \quad \sqrt{42.25} = 6.5$

EXAMPLE 2. *Evaluate* $\sqrt{1.96}$.

Solution Using the division method we may find the square root of the given number as shown below.

```
    1 | 1.96 (1.4
      |-1
   24 |  96
      | -96
      |  ×
```

$\therefore \quad \sqrt{1.96} = 1.4$

EXAMPLE 3. *Evaluate* $\sqrt{6.4009}$.

Solution Using the division method we may find the square root of the given number as shown below.

```
    2 | 6.40 09 (2.53
      |-4
   45 | 240
      |-225
  503 |  1509
      | -1509
      |   ×
```

$\therefore \quad \sqrt{6.4009} = 2.53$

EXAMPLE 4. *Evaluate* $\sqrt{0.4225}$.

Solution Using the division method we may find the square root of the given number as shown below.

```
    6 | 0.42 25 (.65
      |  -36
  125 |   625
      |  -625
      |    ×
```

$\therefore \quad \sqrt{0.4225} = 0.65$

To find the value of square root correct up to certain places of decimal

If the square root is required correct up to two places of decimal, we shall find it up to 3 places of decimal and then round it off up to two places of decimal.

Similarly, if the square root is required correct up to three places of decimal, we shall find it up to 4 places of decimal and then round it off up to three places of decimal, and so on.

EXAMPLE 5. *Evaluate $\sqrt{2}$ correct up to two places of decimal.*

Solution Using the division method we may find the value of $\sqrt{2}$ as shown below.

1	$\overline{2}.\overline{00}\ \overline{00}\ \overline{00}$ (1.414
	−1
24	100
	−96
281	400
	−281
2824	11900
	−11296
	604

$\therefore \quad \sqrt{2} = 1.414 \Rightarrow \sqrt{2} = 1.41$ (correct up to 2 places of decimal)

EXAMPLE 6. *Evaluate $\sqrt{0.8}$ correct up to two places of decimal.*

Solution Using the division method, we may find the value of $\sqrt{0.8}$ as shown below.

8	$0.\overline{80}\ \overline{00}\ \overline{00}$ (0.894
	−64
169	1600
	−1521
1784	7900
	−7136
	764

$\therefore \quad \sqrt{0.80} = 0.894 \Rightarrow \sqrt{0.8} = 0.89$ (correct up to 2 places of decimal)

EXERCISE 3F

Evaluate:

1. $\sqrt{1.69}$
2. $\sqrt{33.64}$
3. $\sqrt{156.25}$
4. $\sqrt{75.69}$
5. $\sqrt{9.8596}$
6. $\sqrt{10.0489}$
7. $\sqrt{1.0816}$
8. $\sqrt{0.2916}$
9. Evaluate $\sqrt{3}$ up to two places of decimal.
10. Evaluate $\sqrt{2.8}$ correct up to two places of decimal.
11. Evaluate $\sqrt{0.9}$ correct up to two places of decimal.
12. Find the length of each side of a square whose area is equal to the area of a rectangle of length 13.6 metres and breadth 3.4 metres.

SQUARE ROOTS OF FRACTIONS

For any positive numbers a and b, we have:

(i) $\sqrt{ab} = (\sqrt{a} \times \sqrt{b})$ *(ii)* $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$.

SOLVED EXAMPLES

EXAMPLE 1. *Evaluate* $\sqrt{\dfrac{441}{961}}$.

Solution We have $\sqrt{\dfrac{441}{961}} = \dfrac{\sqrt{441}}{\sqrt{961}}$.

Now, we find the square roots of 441 and 961, as shown below.

$$\begin{array}{r|l} 2 & \overline{4}\ \overline{41}\ (21 \\ & \underline{-4} \\ 41 & \ \ 41 \\ & \underline{-41} \\ & \ \ \times \end{array} \qquad \begin{array}{r|l} 3 & \overline{9}\ \overline{61}\ (31 \\ & \underline{-9} \\ 61 & \ \ 61 \\ & \underline{-61} \\ & \ \ \times \end{array}$$

Thus, $\sqrt{441} = 21$ and $\sqrt{961} = 31$

$$\Rightarrow \quad \sqrt{\frac{441}{961}} = \frac{\sqrt{441}}{\sqrt{961}} = \frac{21}{31}.$$

EXAMPLE 2. *Find the square root of* $1\dfrac{56}{169}$.

Solution $1\dfrac{56}{169} = \dfrac{225}{169}$.

$$\therefore \quad \sqrt{1\frac{56}{169}} = \sqrt{\frac{225}{169}} = \frac{\sqrt{225}}{\sqrt{169}}.$$

We find the square roots of 225 and 169 separately, as given below.

$$\begin{array}{r|l} 1 & \overline{2}\ \overline{25}\ (15 \\ & \underline{-1} \\ 25 & 125 \\ & \underline{-125} \\ & \ \ \times \end{array} \qquad \begin{array}{r|l} 1 & \overline{1}\ \overline{69}\ (13 \\ & \underline{-1} \\ 23 & \ \ 69 \\ & \underline{-69} \\ & \ \ \times \end{array}$$

$\therefore \quad \sqrt{225} = 15$ and $\sqrt{169} = 13$

$$\Rightarrow \quad \sqrt{1\frac{56}{169}} = \sqrt{\frac{225}{169}} = \frac{\sqrt{225}}{\sqrt{169}} = \frac{15}{13} = 1\frac{2}{13}.$$

EXAMPLE 3. *Find the value of* $\dfrac{\sqrt{243}}{\sqrt{363}}$.

Solution We have

$$\frac{\sqrt{243}}{\sqrt{363}} = \sqrt{\frac{243}{363}} = \sqrt{\frac{81}{121}} = \frac{\sqrt{81}}{\sqrt{121}} = \frac{9}{11}.$$

EXAMPLE 4. *Find the value of* $\sqrt{45} \times \sqrt{20}$.

Solution We have

$$\sqrt{45} \times \sqrt{20} = \sqrt{45 \times 20} = \sqrt{3 \times 3 \times 5 \times 2 \times 2 \times 5}$$
$$= \sqrt{\underline{3 \times 3} \times \underline{2 \times 2} \times \underline{5 \times 5}} = (3 \times 2 \times 5) = 30.$$

EXERCISE 3G

Evaluate:

1. $\sqrt{\frac{16}{81}}$
2. $\sqrt{\frac{64}{225}}$
3. $\sqrt{\frac{121}{256}}$
4. $\sqrt{\frac{625}{729}}$
5. $\sqrt{3\frac{13}{36}}$
6. $\sqrt{4\frac{73}{324}}$
7. $\sqrt{3\frac{33}{289}}$
8. $\frac{\sqrt{80}}{\sqrt{405}}$
9. $\frac{\sqrt{1183}}{\sqrt{2023}}$
10. $\sqrt{98} \times \sqrt{162}$

EXERCISE 3H

OBJECTIVE QUESTIONS

Tick (✓) the correct answer in each of the following:

1. Which of the following numbers is not a perfect square?
   (a) 7056 (b) 3969 (c) 5478 (d) 4624
   Hint. *The number* 5478 *ends in* 8.
2. Which of the following numbers is not a perfect square?
   (a) 1444 (b) 3136 (c) 961 (d) 2222
   Hint. *The number* 2222 *ends in* 2.
3. Which of the following numbers is not a perfect square?
   (a) 1843 (b) 3721 (c) 1024 (d) 1296
   Hint. *The number* 1843 *ends in* 3.
4. Which of the following numbers is not a perfect square?
   (a) 1156 (b) 4787 (c) 2704 (d) 3969
   Hint. *The number* 4787 *ends in* 7.
5. Which of the following numbers is not a perfect square?
   (a) 3600 (b) 6400 (c) 81000 (d) 2500
   Hint. *The number* 81000 *ends in an odd number of zeros.*
6. Which of the following cannot be the unit digit of a perfect square number?
   (a) 6 (b) 1 (c) 9 (d) 8
7. The square of a proper fraction is
   (a) larger than the fraction (b) smaller than the fraction
   (c) equal to the fraction (d) none of these
8. If n is odd, then $(1+3+5+7+\dots$ to n terms) is equal to
   (a) (n^2+1) (b) (n^2-1) (c) n^2 (d) $(2n^2+1)$
9. Which of the following is a Pythagorean triplet?
   (a) (2, 3, 5) (b) (5, 7, 9) (c) (6, 9, 11) (d) (8, 15, 17)
10. What least number must be subtracted from 176 to make it a perfect square?
    (a) 16 (b) 10 (c) 7 (d) 4
11. What least number must be added to 526 to make it a perfect square?
    (a) 3 (b) 2 (c) 1 (d) 6

12. What least number must be added to 15370 to make it a perfect square?

(a) 4 (b) 6 (c) 8 (d) 9

13. $\sqrt{0.9} = ?$

(a) 0.3 (b) 0.03 (c) 0.33 (d) 0.94

14. $\sqrt{0.1} = ?$

(a) 0.1 (b) 0.01 (c) 0.316 (d) none of these

15. $\sqrt{0.9} \times \sqrt{1.6} = ?$

(a) 0.12 (b) 1.2 (c) 0.75 (d) 12

Hint. $\sqrt{0.9} \times \sqrt{1.6} = \sqrt{1.44} = 1.2$.

16. $\dfrac{\sqrt{288}}{\sqrt{128}} = ?$

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{3}{\sqrt{2}}$ (c) $\dfrac{3}{2}$ (d) 1.49

Hint. $\dfrac{\sqrt{288}}{\sqrt{128}} = \sqrt{\dfrac{288}{128}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$.

17. $\sqrt{2\dfrac{1}{4}} = ?$

(a) $2\dfrac{1}{2}$ (b) $1\dfrac{1}{2}$ (c) $1\dfrac{1}{4}$ (d) none of these

Hint. $\sqrt{2\dfrac{1}{4}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2} = 1\dfrac{1}{2}$.

18. Which of the following is the square of an even number?

(a) 196 (b) 441 (c) 625 (d) 529

19. Which of the following is the square of an odd number?

(a) 2116 (b) 3844 (c) 1369 (d) 2500

Things to Remember

1. *The square of a number is the product of the number with the number itself. Thus, square of $x = (x \times x)$, denoted by x^2.*
2. *A natural number n is a perfect square if $n = m^2$ for some natural number m.*
3. *A number ending in 2, 3, 7 or 8 is never a perfect square.*
4. *A number ending in an odd number of zeros is never a perfect square.*
5. (i) *The square of an even number is even.*
   (ii) *The square of an odd number is odd.*
6. *For any natural number n, we have*
   n^2 = *(sum of the first n odd natural numbers).*
7. *For any natural number m greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.*
8. *The square root of a number x is that number which when multiplied by itself gives x as the product, and we denote the square root of x by $\sqrt{x}$.*
9. *In order to find the square root of a perfect square, resolve it into prime factors; make pairs of similar factors, and take the product of prime factors, choosing one out of every pair.*

10. *For finding the square root of a decimal fraction, make the number of decimal places even by affixing a zero, if necessary; mark the periods, and find out the square root, putting the decimal point in the square root as soon as the integral part is exhausted.*
11. *For positive numbers a and b, we have:*

(i) $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

SOME INTERESTING PATTERNS (JUST FOR FUN)

PATTERN 1.

$(11)^2 = 121$
$(101)^2 = 10201$
$(1001)^2 = 1002001$
$(10001)^2 = \ldots\ldots$
$(100001)^2 = \ldots\ldots$
$(1000001)^2 = \ldots\ldots$
$(10000001)^2 = \ldots\ldots$

Solution Clearly, we have:

$(10001)^2 = 100020001$
$(100001)^2 = 10000200001$
$(1000001)^2 = 1000002000001$
$(10000001)^2 = 100000020000001$

PATTERN 2.

$(11)^2 = 121$
$(101)^2 = 10201$
$(10101)^2 = 102030201$
$(1010101)^2 = 1020304030201$
$(101010101)^2 = \ldots\ldots$

Solution Clearly, we have 5 ones. So, we take counting up to 5 and then in reverse order putting 0 in between each pair.

$\therefore$ $(101010101)^2 = 10203040504030201.$

PATTERN 3.

$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + (12)^2 = (13)^2$
$4^2 + 5^2 + \ldots\ldots = \ldots\ldots$
$5^2 + 6^2 + \ldots\ldots = \ldots\ldots$
$6^2 + \ldots\ldots + \ldots\ldots = \ldots\ldots$

Solution We have $a^2 + b^2 + c^2 = (c+1)^2$, where $c = a \times b$.

$\therefore$ $4^2 + 5^2 + (20)^2 = (21)^2$
$5^2 + 6^2 + (30)^2 = (31)^2$
$6^2 + 7^2 + (42)^2 = (43)^2$

PATTERN 4.

$(1)^2 = 1$
$(11)^2 = 121$
$(111)^2 = 12321$
$(1111)^2 = 1234321$
$(11111)^2 = 123454321$
$(111111)^2 = 12345654321$
$(1111111)^2 = \ldots\ldots\ldots\ldots\ldots\ldots$

Solution In the given number we have 8 ones, so we write numbers from 1 to 8 and then in reverse order up to 1.

$\therefore \quad (11111111)^2 = 123456787654321.$

PATTERN 5. $(7)^2 = 49$

$(67)^2 = 4489$

$(667)^2 = 444889$

$(6667)^2 = 44448889$

$(66667)^2 = 4444488889$

$(666667)^2 = \ldots\ldots\ldots\ldots\ldots\ldots$

Solution Clearly, we have (5 sixes and one seven).

So, the square must have 6 fours 5 eights and 1 nine.

$\therefore \quad (666667)^2 = 444444888889.$

TEST PAPER-3

A. **1.** Evaluate $\sqrt{11236}$.

2. Find the greatest number of five digits which is a perfect square. What is the square root of this number?

3. Find the least number of four digits which is a perfect square. What is the square root of this number?

4. Evaluate $\sqrt{0.2809}$.

5. Evaluate $\sqrt{3}$ correct up to two places of decimal.

6. Evaluate $\frac{\sqrt{48}}{\sqrt{243}}$.

B. ***Mark (✓) against the correct answer in each of the following:***

7. Which of the following numbers is not a perfect square?

(a) 529 (b) 961 (c) 1024 (d) 1222

8. $\sqrt{2\frac{1}{4}} = ?$

(a) $2\frac{1}{2}$ (b) $1\frac{1}{4}$ (c) $1\frac{1}{2}$ (d) none of these

9. Which of the following is the square of an even number?

(a) 529 (b) 961 (c) 1764 (d) 2809

10. What least number must be added to 521 to make it a perfect square?

(a) 3 (b) 4 (c) 5 (d) 8

11. What least number must be subtracted from 178 to make it a perfect square?

(a) 6 (b) 8 (c) 9 (d) 7

12. $\sqrt{72} \times \sqrt{98} = ?$

(a) 42 (b) 84 (c) 64 (d) 74

C. **13.** ***Fill in the blanks.***

(i) $1 + 3 + 5 + 7 + 9 + 11 + 13 = (......)^2$.

(ii) $\sqrt{1681} =$ ....

(iii) The smallest square number exactly divisible by 2, 4, 6 is ....

(iv) A given number is a perfect square having n digits, where n is odd. Then, its square root will have ...... digits.